

WJEC GCSE **HIGHER**

MATHEMATICS AND NUMERACY

Answers

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MADE FOR
WALES

Section 1: Number

Chapter 1.1 Numbers and rounding

Now try these (p 6)

- (a) one thousand, five hundred and five

(b) fifteen thousand and twenty-five

(c) forty-five thousand and twelve

(d) two hundred and thirty-five thousand, eight hundred and ninety-one

(e) three million, five hundred and sixty thousand and nine

(f) two million, one hundred thousand and fifty
- (a) 3501

(b) 7019

(c) 40 891

(d) 105 210

(e) 5 050 010
- (a) 168

(b) one hundred and sixty-eight

Now try these (p 7)

- (a) -4 (e) -3

(b) -10 (f) -6

(c) 6 (g) -17

(d) -6 (h) -4
- $35.2\text{ }^{\circ}\text{C} - (-23.3\text{ }^{\circ}\text{C}) = 58.5\text{ }^{\circ}\text{C}$

- (a) -27 (d) -48

(b) -42 (e) -54

(c) -21 (f) 96
- (a) 7 (d) 6

(b) -9 (e) -32

(c) -7 (f) 8

Now try these (p 8-9)

- (a) 1 (c) 3

(b) 1 (d) 14
- (a) (i) 1250 (ii) 1300

(b) (i) 180 (ii) 200

(c) (i) 150 (ii) 100

(d) (i) 10 (ii) 0
- (a) 372 000

(b) 370 000

(c) 400 000
- (a) Monday: 460

Tuesday: 510

Wednesday: 610

Thursday: 770

Friday: 900

Saturday: 1040

Sunday: 1490

(b) Monday: 500

Tuesday: 500

Wednesday: 600
 Thursday: 800
 Friday: 900
 Saturday: 1000
 Sunday: 1500

Now try these (p 10)

- (a) 0.6 (e) 1.64
 (b) 0.03 (f) 664.3
 (c) 0.786 (g) 23.755
 (d) 10.78 (h) 63.24
- (a) incorrect, 0.000
 (b) correct
 (c) correct
 (d) correct
 (e) incorrect, 0.005
- (a) 10 (e) 1
 (b) 100 (f) 34.8
 (c) 0.79 (g) 16 800
 (d) 12.8 (h) 77
- (a) 0.005 (e) 0.1
 (b) 0.0066 (f) 0.35
 (c) 0.09 (g) 0.988
 (d) 0.001 (h) 0.0011

Now try these (p 11)

- (a) $0.4 \times 46 = 18.4 =$ maximum 18 cattle
 (b) $0.8 \times 46 = 36.8 =$ maximum 36 cattle
- (a) $1000 \div 30 = 33.333333... \approx 34$ (rounded up to account for the leftover apples)

(b) $1000 \div 60 = 16.66666... \approx 17$ (rounded up to account for the leftover apples)

Now try these (p 12)

- (a) $27 \times 19 \approx 30 \times 20 = 600$
 (b) $102 \times 17 \approx 100 \times 20 = 2000$
 (c) $45 \times 55 \approx 50 \times 60 = 3000$
 (d) $72 \times 89 \approx 70 \times 90 = 6300$
 (e) $13 \times 909 \approx 10 \times 900 = 9000$
 (f) $13 \times 999 \approx 10 \times 1000 = 10\,000$
- (a) $3.45 \times 7.60 \approx 3 \times 8 = 24$
 (b) $45 \times 89 \approx 50 \times 90 = 4500$
 (c) $123 \times 117 \approx 120 \times 120 = 14\,400$
 (d) $23.4 \div 4.8 \approx 20 \div 5 = 4$
 (e) $83.9 \div 3.99 \approx 84 \div 4.0 = 21$
 (f) $75.3 \div 4.96 \approx 75 \div 5.0 = 15$
- $\approx \frac{2400}{60}$, as the numbers have been rounded the least.
- (a) $\sqrt{26} \approx \sqrt{25} = 5$
 (b) $6.3^2 \approx 6^2 = 36$
 (c) $\sqrt{148} \approx \sqrt{144} = 12$
 (d) $3.5 \div 7.1 \approx 3.5 \div 7 = 0.5$
 (e) $\frac{20}{2.4+1.4} \approx \frac{20}{2.5+1.5} = \frac{20}{4} = 5$
 (f) $\frac{\sqrt{48}}{6.5} \approx \frac{\sqrt{49}}{7} = \frac{7}{7} = 1$

Now try these (p 13)

- (a) $3.45 + 2.78 + 0.09 \approx 3 + 3 + 0 = 6$ **Answer B**
 (b) $12.56 - 1.87 - 0.45 \approx 13 - 2 - 0.5 = 10.5$ **Answer C**

(c) $120 \div 0.45 \approx 100 \div 0.5$

= 200 **Answer B**

Note: student may also use

$120 \div 0.5 = 240$ to arrive at the same answer

(d) $0.01 \times 0.15 \times 109 \approx 0.01 \times 0.2 \times 100$

= 0.2 **Answer B**

(e) $(0.12 + 300) \times 0.53 \approx (0 + 300) \times 0.53$

= 159 **Answer A**

Note: student may also use

$300 \times 0.5 = 150$ to arrive at the same answer

(f) $6.07 + (3.67 \times 0.1) \approx 6 + (4 \times 0.1)$

= 6.4 ≈ 6 **Answer A**

(g) $20.75 \div 6.98 \div 2.1 \approx 21 \div 7 \div 2 = 3 \div 2$

= 1.5 **Answer A**

(h) $0.01 \times 145 \times 35 \approx 0.01 \times 100 \times 40$

= 40 **Answer A**

(i) $6.5 \times 0.3 \times 0.01 \approx 7 \times 0.3 \times 0.01 = 0.021$

≈ 0.02 **Answer B**

(j) $65 \div (0.15 \times 1050) \approx 70 \div (0.2 \times 1000) =$

$70 \div 20 = 3.5 \approx 4$ **Answer C**

2. $(301 + 479) \times 8 \approx (300 + 500) \times 8 = 6400$

Bryn is incorrect by a factor of 10 so has probably multiplied by 80 instead of 8.

Chapter 1.2 Properties of numbers

Now try these (p 14)

1. (a) 49 (d) 84

(b) 68 (e) 5

(c) 9 (f) 16

2. 5

3. (a) 62.50 (b) 166.50

(c) 1.78

Now try these (p 16)

1. (a) $56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$

(b) $160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^5 \times 5$

(c) $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

(d) $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3$

2. $850 = 2 \times 5 \times 5 \times 17 = 2 \times 5^2 \times 17$

3. $122 = 2 \times 61$, $244 = 2 \times 2 \times 61 = 2^2 \times 61$
Since 244 is double 122, their expressions as prime factors are the same, except 244 has an extra 2

Now try these (p 17)

1. (a) $900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 2^2 \times 3^2 \times 5^2$

(b) Yes, since when you multiply square numbers together, the answer will always be a square number.

(c) $\sqrt{900} = \sqrt{2^2 \times 3^2 \times 5^2} = 2 \times 3 \times 5 = 30$

2. (a) $480 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^5 \times 3 \times 5$

(b) 480 is not a square number as its prime factors are not in even powers.

3. 2 because $2^3 \times 2 = 2^4$, meaning both terms in the calculation will have even powers and are therefore square numbers.

4. (a) No, one of the prime factors does not have an even power in index form.

(b) Yes

(c) Yes

(d) No, one of the prime factors does not have an even power in index form.

5. 3 because this is the smallest of the prime factors.

6. $\sqrt{3 \times 3 \times 5 \times 5 \times 7 \times 7} = \sqrt{3^2 \times 5^2 \times 7^2} = 3 \times 5 \times 7 = 105$

Now try these (p 19)

- (a) 1, 2, 4, 8, 16, 32
(b) 1, 2, 4, 8, 10, 20, 40, 80
(c) 8
- Highest common factor = 5, lowest common multiple = 140
- (a) $45 = 3^2 \times 5$
(b) $78 = 2 \times 3 \times 13$
(c) $\text{LCM} = 2 \times 3 \times 3 \times 5 \times 13 = 1170$
(d) $90 = 2 \times 3^2 \times 5$, $156 = 2^2 \times 3 \times 13$
 $\text{LCM} = 2 \times 2 \times 3 \times 3 \times 5 \times 13 = 2340$
The numbers have doubled, and so has their lowest common multiple.
- (a) 29, 37 (c) 45
(b) 45 (d) 26
- Start with the list of square numbers whose doubles are between 1 and 100 to account for "giving a square number when halved":
1, 4, 9, 16, 25, 36, 49
If half a number has a factor of 7, so does the original number, so the square number we need is 49 and therefore the number that meets all of the criteria is **98**
- (a) $\frac{1}{3}$ (g) $\frac{1}{2}$
(b) $\frac{1}{4}$ (h) 2
(c) $\frac{1}{7}$ (i) 4
(d) 5 (j) 10
(e) 10 (k) 2
(f) 3 (l) 8
- (a) $3.5 = \frac{7}{2}$, so reciprocal = $\frac{2}{7}$
(b) $1.5 = \frac{3}{2}$, so reciprocal = $\frac{2}{3}$
(c) $2.25 = \frac{9}{4}$, so reciprocal = $\frac{4}{9}$

- (d) $3.2 = \frac{16}{5}$, so reciprocal = $\frac{5}{16}$
(e) $2\frac{1}{2} = \frac{5}{2}$, so reciprocal = $\frac{2}{5}$
(f) $3\frac{1}{8} = \frac{25}{8}$, so reciprocal = $\frac{8}{25}$
- $0.5^2 = 0.25 = \frac{1}{4}$ so $\frac{1}{0.5^2} = \frac{1}{\frac{1}{4}} = 4$
- The reciprocal of 100 = $\frac{1}{100} = 0.01$
- The reciprocal of 200 = $\frac{1}{200} = 0.005$

Chapter 1.3 Indices

Now try these (p 21)

- (a) 2^6 (c) 17^3
(b) 5^5 (d) 3^6
- (a) 2^6 (f) 7^{15}
(b) 7^2 (g) 8^5
(c) 4^5 (h) 2^{24}
(d) 7^7 (i) 5^{25}
(e) 2^6 (j) 6^{24}
- (a) x^9 (f) y^8
(b) x^3 (g) x^8
(c) c^6 (h) a^6
(d) g^9 (i) q^4
(e) p^4 (j) m^{-7}
- (a) $a^1 = a$ (b) y^5
- $7^8 \times 7^{28} = 7^{36}$
 $7^{36} \div 7^6 = 7^{30}$, so $n = 30$
- (a) $8 \times 4 = 32$ (d) $3^3 = 27$
(b) $4^2 = 16$ (e) $18^2 = 324$
(c) $4^3 = 48$ (f) $2^6 = 64$
- $3^{17} \times 3^2 = 3^{19}$
 $3^{19} \div 3^5 = 3^{14}$, so $n = 14$

8. $5^{27} \div 5^{16} = 5^{11}$
 $5^{17} \div 5^{11} = 5^6$, so $n = 6$
9. (a) $3^2 = 9$ and $2^2 = 4$, so
 $3^2 \times (2^2)^2 = 9 \times 4^2 = 9 \times 16 = 144$
- (b) $5^{30} \times 5^4 = 5^{34}$
 $5^{34} \div 5^{32} = 5^2 = 25$
- (c) $4^{10} \times 4^{10} = 4^{20}$
 $4^{20} \div 4^{18} = 4^2 = 16$
- (d) $(3^2)^3 = 9^3 = 729$
10. $9 = 3^2$ and $27 = 3^3$, so
 $(9^3)^2 \div 27 = ((3^2)^3)^2 \div 3^3 = 3^{12} \div 3^3 = 3^9$,
so $n = 9$
11. $4 = 2^2$, so
 $(4^5)^3 \div 4^7 = ((2^2)^5)^3 \div (2^2)^7 = 2^{30} \div 2^{14} = 2^{16}$,
so $n = 16$
12. $(x^5 \times x^3)^2 = (x^8)^2 = x^{16}$, so
 $(x^5 \times x^3)^2 \div x^{10} = x^{16} \div x^{10} = x^6$, so $n = 6$
13. $(x^9 \div x^4)^2 = (x^5)^2 = x^{10}$, so
 $(x^9 \div x^4)^2 \div x^{10} = x^{10} \div x^{10} = x^0$, so $n = 0$
This can also be written as 1

Now try these (pp 22)

1. (a) $\frac{1}{2^3}$ (d) $\frac{1}{3^3}$
(b) $\frac{1}{5^4}$ (e) $\frac{1}{12^7}$
(c) $\frac{1}{7^6}$ (f) $\frac{1}{24^{12}}$
2. (a) 8 (d) 16
(b) 2 (e) $\frac{3}{2}$
(c) 1 (f) $27 + 9 = 36$
3. (a) $\frac{25}{\frac{1}{2}} = 50$
(b) $\left(\frac{1}{3}\right)^2 \times 1 = \frac{1}{9}$

4. $\frac{2 \times 8}{2^{-7}} = \frac{2 \times 2^3}{2^{-7}} = \frac{2^4}{2^{-7}}$
 $= 2^{4-(-7)} = 2^{11}$ or 2048
5. $\left((3^2)^4\right)^2 \times 27^{-8}$
 $= 3^{16} \times (3^3)^{-8}$
 $= 3^{16} \times 3^{-24} = 3^{-8}$

Now try these (pp 25)

1. (a) 1×10^3 (d) 8.6×10^{-2}
(b) 1.86×10^5 (e) 1.5×10^{-5}
(c) 1.7×10^6 (f) 9.031×10^{-4}
2. (a) 1500 (d) 0.5
(b) 600 500 (e) 0.000609
(c) 250 000 000 (f) 0.0000001
3. (a) $3200 = 3.2 \times 10^3$
(b) $6999.999998 = 6.999999998 \times 10^3$
(c) $30 \times 10^6 = 3.0 \times 10^7$
(d) 4×10^1
(e) $27 \times 10^5 = 2.7 \times 10^6$
(f) 1.25×10^6
4. $1000 \div (3.27 \times 10^{-22})$
 $= (1 \times 10^3) \div (3.27 \times 10^{-22})$
 $= \frac{1}{3.27} \times 10^{3-(-22)}$
 $= \frac{1000}{327} \times 10^{25}$
5. $4 \div (2 \times 10^{-2})$
 $= (4 \times 10^0) \div (2 \times 10^{-2})$
 $= 2 \times 10^2$ or 200

Chapter 1.4

Fractions, decimals, percentages and ratios

Now try these (pp 27–28)

- (a) $\frac{1}{2}$ and $\frac{4}{8}$

(b) $\frac{24}{36}$ and $\frac{2}{3}$

(c) $\frac{3}{8}$ and $\frac{9}{24}$
- (a) $\frac{4}{5}$ (d) $\frac{31}{35}$

(b) $\frac{2}{7}$ (e) $\frac{10}{11}$

(c) $\frac{5}{7}$ (f) $\frac{36}{107}$
- (a) $\frac{3}{8} = \frac{\boxed{15}}{40} = \frac{\boxed{12}}{32} = \frac{9}{\boxed{24}}$

(b) $\frac{20}{30} = \frac{\boxed{10}}{15} = \frac{4}{\boxed{6}} = \frac{2}{\boxed{3}}$

(c) $\frac{15}{120} = \frac{\boxed{5}}{40} = \frac{3}{\boxed{24}} = \frac{30}{\boxed{240}}$
- (a) $\frac{0.5}{2} = \frac{5}{20} = \frac{1}{4}$

(b) $\frac{44.5}{50} = \frac{445}{500} = \frac{89}{100}$

(c) $\frac{32.5}{40} = \frac{325}{400} = \frac{13}{16}$

Now try these (p 29)

- (a) $\frac{18}{5}$

(b) $1\frac{5}{3}$

(c) $\frac{1}{4}$

- (a) $4\frac{4}{15}$ (c) $6\frac{1}{2}$

(b) $10\frac{1}{3}$ (d) $2\frac{13}{16}$
- (a) $\frac{8}{3}$ (c) $\frac{47}{6}$

(b) $\frac{23}{5}$ (d) $\frac{139}{11}$
- $\frac{21}{2}$
- $5\frac{3}{8}$

Now try these (p 30)

- Ash's score on the Welsh test is $\frac{3}{4}$, which is equivalent to $\frac{15}{20}$ which is less than $\frac{16}{20}$, so Ash got the higher score in the French test.
- (a) $\frac{5}{16} + \frac{7}{16} = \frac{12}{16} = \frac{3}{4}$

(b) $\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$

(c) $1\frac{3}{4} + 2\frac{3}{5} = 3 + \frac{3}{4} + \frac{3}{5} = 3 + \frac{15}{20} + \frac{12}{20} = 3 + \frac{27}{20} = 3 + 1\frac{7}{20} = 4\frac{7}{20}$

(d) $\frac{2}{5} - \frac{1}{3} = \frac{6}{15} - \frac{5}{15} = \frac{1}{15}$

(e) $\frac{5}{7} - \frac{3}{8} = \frac{40}{56} - \frac{21}{56} = \frac{19}{56}$

(f) $2\frac{3}{4} - 1\frac{1}{3} = 1 + \frac{3}{4} - 1 + \frac{1}{3} = 1 + \frac{9}{12} - \frac{4}{12} = 1\frac{5}{12}$

Now try these (pp 31–32)

- (a) $\frac{27}{40}$

(b) $16 \div \frac{4}{5} = 16 \times \frac{5}{4} = \frac{16 \times 5}{4} = \frac{80}{4} = 20$

(c) $24 \times \frac{8}{15} = \frac{24 \times 8}{15} = \frac{192}{15} = \frac{64}{5} = 12\frac{4}{5}$

$$(d) \frac{4}{5} \div 8 = \frac{4}{5 \times 8} = \frac{4}{40} = \frac{1}{10}$$

$$(e) \frac{3}{25} \div 3^2 = \frac{3}{25 \div 9} = \frac{3}{25 \times 9} = \frac{3}{225} = \frac{1}{75}$$

$$(f) \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1 \times 2 \times 3}{2 \times 3 \times 4} = \frac{6}{24} = \frac{1}{4}$$

$$2. \quad 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$3. \quad (a) 728$$

$$(b) 185$$

$$4. \quad \frac{42}{260} = \frac{21}{130}$$

$$5. \quad \left(\frac{6}{36}\right) \div 3 = \left(\frac{6}{36}\right) \times \frac{1}{3} = \frac{1}{18}$$

$$2. \quad (a) \frac{125 \text{ g}}{12 \text{ kg}} \times 100 = \frac{125 \text{ g}}{12\,000 \text{ g}} \times 100$$

$$= \frac{125}{12\,000} \times 100 = \frac{1.041\dot{6}}{100} \times 100$$

$$= 1.04\% \text{ (to 2 d.p.)}$$

$$(b) \frac{14}{60} \times 100 = \frac{23.\dot{3}}{100} \times 100 = 23.\dot{3}\%$$

$$(c) \frac{35 \text{ minutes}}{2 \text{ hours } 20 \text{ minutes}} = \frac{35 \text{ minutes}}{140 \text{ minutes}}$$

$$= \frac{1}{4} = 25\%$$

$$3. \quad 100\% - 20\% = 80\%$$

80% of £450 = £360 left to pay.

£360 ÷ 12 months = £30 each month.

Now try these (p 32)

$$1. \quad (a) \frac{1}{8} \text{ of } 240 = 30$$

$$(b) \frac{2}{9} \text{ of } £108 = £24$$

$$(c) \frac{4}{10} \text{ of } 1010 \text{ m} = 404 \text{ m}$$

$$(d) \frac{13}{20} \text{ of } 2400 \text{ ml} = 1560 \text{ ml}$$

$$(e) \frac{3}{5} \text{ of } 145 \text{ g} = 87 \text{ g}$$

$$(f) \frac{7}{8} \text{ of } 32 \text{ km} = 28 \text{ km}$$

$$2. \quad \frac{3}{8} \text{ of } 1 \text{ km} = \frac{3}{8} \text{ of } 1000 \text{ m}$$

$$= 375 \text{ m}$$

$$3. \quad \frac{3}{4} \text{ of } 26.2 \text{ miles} = 19.65 \text{ miles}$$

Now try these (pp 34–35)

$$1. \quad (a) 20\% \text{ of } 240 = (240 \div 100) \times 20 = 48$$

$$(b) 15\% \text{ of } 190 = (190 \div 100) \times 15 = 28.5$$

$$(c) 72\% \text{ of } 55 = (55 \div 100) \times 72 = 39.6$$

$$(d) 32.5\% \text{ of } 150 = (150 \div 100) \times 32.5$$

$$= 48.75$$

Now try these (p 36)

$$1. \quad \frac{440 - 400}{400} \times 100 = \frac{40}{400} \times 100 = 10\% \text{ increase}$$

$$2. \quad \frac{1100000 - 600000}{600000} \times 100 = \frac{500000}{600000} \times 100$$

$$= \frac{83.\dot{3}}{100} \times 100 = 83.3\% \text{ increase (1 d.p.)}$$

$$3. \quad \frac{25000 - 8000}{25000} \times 100 = \frac{17000}{25000} \times 100$$

$$= \frac{68}{100} \times 100 = 68\% \text{ decrease}$$

$$4. \quad \frac{8000 - 4500}{4500} \times 100 = \frac{3500}{4500} \times 100 = \frac{77.\dot{7}}{100} \times 100$$

$$= 77.8\% \text{ increase (1 d.p.)}$$

5.

| Cost before | Cost after | % change |
|-------------|--------------|------------------------------|
| £2 | £3 | 50% increase |
| £12.50 | £17.45 | 39.6% increase |
| £109 | £117 | 7.3% increase (to 1 d.p.) |
| £10 000 | £12 075 | 20.75% increase |
| £750 000 | £1.2 million | 60% increase |

Now try these (p 37)

- (a) $125 \text{ cm} \times 1.17 = 146.25 \text{ cm}$
(b) $£55 \times 1.06 = £58.30$
(c) $\$400 \times 1.085 = \434
(d) $£15\,000 \times 1.046 = £15\,690$
(e) $£15\,000 \times 1.0578 = £15\,867$
(f) $12.5 \text{ km} \times 1.821 = 22.7625 \text{ km}$
- (a) $25 \text{ m} \times 0.94 = 23.5 \text{ m}$
(b) $£340 \times 0.91 = £309.40$
(c) $30 \text{ m} \times 0.87 = 26.1 \text{ m}$
(d) $£34\,000 \times 0.855 = £29\,070$
(e) $5 \text{ km} \times 0.903 = 4.515 \text{ km}$
(f) $1750 \text{ m} \times 0.875 = 1531.25 \text{ m}$
- $£27\,000 \times 0.83 = £22\,410$
- $145\,000 \times 1.095 = 158\,775$
- $£1\,200\,000 \times 0.07 = £84\,000$
- $£150 \times 1.1 = £165$
 $£165 \times 0.95 = £156.75$

Now try these (p 39)

- £2
- £900
- 40
- (a) £10 600
(b) £2650

Now try these (p 40)

- (a) $A = 400 \times 0.92^3$
(b) B = decrease of 8% each year over 2 years
C = increase of 8% each year over 3 years

D = decrease of 92% each year over 3 years

- (a) $£30\,000 \times 1.048^2 = £32\,949.12$
(b) $£30\,000 \times 1.048^4 - £30\,000 = £6188.15$
- (a) $£2000 \times 1.057^3 = £2361.86$
(b) $£2000 \times 1.057^5 = £2638.79$
- $2 \text{ m} \times 1.2^3 = 3.456 \text{ m}^2$
- (a) $£27\,000 \times 0.78 = £21\,060$
(b) $£21\,060 \times 0.85^4 = £10\,993.45$
 $£27\,000 - £10\,993.45 = £16\,006.55$

Now try these (p 41)

- (a) 160 mm
(b) £67.50
- 600 ml
- (a) 10 648 employees
(b) $\frac{2648}{8000} = \frac{331}{1000}$

Now try these (p 42)

- 0.13, 0.225, 0.25, 0.3, 0.33
- 0.03, 0.3, 0.303, 0.33, 0.333
- $\frac{7}{10}$, 0.7001, 70.1%, 0.71, $\frac{71}{10}$
- $\frac{2}{3} = 66.\dot{6}\%$, $\frac{3}{4} = 75\%$, $\frac{7}{8} = 87.5\%$, $\frac{1}{2} = 50\%$,
 $\frac{7}{12} = 58.\dot{3}\%$

So the fractions in ascending order are:

- $\frac{1}{2}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$
- 89% , $\frac{3}{4}$, 70% , $\frac{2}{3}$, 0.65

Now try these (pp 43–44)

1. (a) 3 : 4 (e) 1 : 4
 (b) 16 : 3 (f) 2 : 5
 (c) 4 : 5 : 9 (g) 10 : 1
 (d) 1 : 10 (h) 4 : 5
2. (a) 4 : 1 (d) 1 : 5
 (b) 5 : 12 (e) 5 : 2
 (c) 3 : 1

Now try these (p 45)

1. £8 and £10
2. (a) 1500 ml = 1.5 l
 (b) 33. $\dot{3}$ ml
3. $85 : 110 = 17 : 22$
4. $400 : 1100 = 4 : 11 = 1 : 2.75$
5. (a) If 10 counters is the difference between the two colours, this is one part, and there are 5 parts in total so the number of counters is $5 \times 10 = 50$ counters.
 (b) $30 : 20 : 10 = 3 : 2 : 1$

Now try these (p 47)

1. (a) 270 g (b) 12.5 (c) $8.727\dots \approx 9$ days
2. $L = 64$
3. $b = 5$
4. (a) 4.5 months
 (b) $\times 1.6$ $\left\{ \begin{array}{l} 5 \text{ workers} \quad 9 \text{ months} \\ 8 \text{ workers} \quad 5.625 \text{ months} \end{array} \right. \div 1.6$
 (c) 1 worker takes 45 months to build a house, so 1 worker would take 900 months to build 20 houses.
 $\times 75$ $\left\{ \begin{array}{l} 1 \text{ worker} \quad 900 \text{ months} \\ 75 \text{ workers} \quad 12 \text{ months} \end{array} \right. \div 75$
5. $x = 9$

Chapter 1.5 Calculations

Now try these (p 49)

1. (a) 3 (c) 1
 (b) 6 (d) $5\frac{1}{2}$ or $\frac{11}{2}$
2. (a) 9 (b) 6

Now try these (p 50)

1. (a) 6 (e) 125
 (b) 12 (f) 2
 (c) 5 (g) 12
 (d) 20 (h) 35
2. (a) 5.75 (c) 10 080
 (b) 5001.142805 (d) 5.21

Now try these (p 52)

1. (a) 782 (g) 477.66
 (b) 2.9 (h) 10.2
 (c) 19.16 (i) 1697
 (d) 33 (j) 0.07
 (e) 787 (k) 0.369
 (f) 16.74 (l) 3.61

Now try these (p 55)

1. (a) 45 000 (g) 290
 (b) 1000 (h) 52 000
 (c) 5313 (i) 11.76
 (d) 15 228 (j) 0.078
 (e) 7303 (k) 0.006
 (f) 24.29 (l) 0.3654

2. (a) 10 (g) $22\frac{1}{3} = 22.\dot{3}$
 (b) 4 (h) $14\frac{2}{9} = 14.\dot{2}$
 (c) 500 (i) 110
 (d) 500 (j) 124
 (e) 8 (k) 208.76
 (f) 7.5 (l) 210.03 (to 2 d.p.)

Now try these (p 57)

1. (a) 8.74 (to 2 d.p.) (g) 8.63 (to 2 d.p.)
 (b) 4.63 (to 2 d.p.) (h) 15.69 (to 2 d.p.)
 (c) -7.70 (to 2 d.p.) (i) 3.99×10^{45} (2 d.p.)
 (d) 19.72 (to 2 d.p.) (j) 0.01 (to 2 d.p.)
 (e) 6.00 (to 2 d.p.) (k) -3.40 (to 2 d.p.)
 (f) 1.07×10^{10} (2 d.p.) (l) 428 663.68 (2 d.p.)
2. (a) 2.2×10^{-3} (to 2 s.f.)
 (b) 3.2×10^{-14} (to 2 s.f.)
 (c) 8.0×10^{12} (to 2 s.f.)
 (d) 3.6×10^{-43} (to 2 s.f.)
 (e) 9.7×10^{-1} (to 2 s.f.)
 (f) 1.0×10^0 (to 2 s.f.)
3. (a) 24.62 (to 2 d.p.) (e) 75.52 (to 2 d.p.)
 (b) 60 (f) 9.46 (to 2 d.p.)
 (c) 56.31 (to 2 d.p.) (g) 48.59 (2 d.p.)
 (d) 8.85 (to 2 d.p.) (h) 56.94 (to 2 d.p.)
4. (a) 2.29 (to 2 d.p.) (e) 2.61 (to 2 d.p.)
 (b) 2.30 (to 2 d.p.) (f) 6.62 (to 2 d.p.)
 (c) 2.89 (to 2 d.p.) (g) 8.34 (2 d.p.)
 (d) 7.73 (to 2 d.p.) (h) 21.30 (to 2 d.p.)
5. (a) 32.20 (to 2 d.p.) (c) 4.10 (to 2 d.p.)
 (b) 7.71 (to 2 d.p.) (d) 28.96 (to 2 d.p.)

Chapter 1.6

Accuracy in rounding

Now try these (p 58)

1. (a) 149.5 cm
 (b) 1505 mm
2. (a) 54.5 mm
 (b) The screw *could* potentially fit, as the greatest possible depth of the hole is 5.5 cm and the shortest possible length of the screw is 5.45 mm.

Now try these (p 59)

1. (a) length: upper = 30.5 m, lower = 29.5 m
 width: upper = 14.05 m, lower = 13.95 m
 (b) length: upper = 5.55 m, lower = 5.45 m
 width: upper = 2.55 m, lower = 2.45 m
 (c) minimum area of patch = 5.45×2.45
 $= 13.3525 \text{ m}^2$
 maximum area of garden = 30.5×14.05
 $= 428.525 \text{ m}^2$
 min. % area of patch = $\frac{13.3525}{428.525} \times 100$
 $= 3.11\%$
 So yes, the patch is definitely takes up more than 3% of Beatriz's garden.
2. (a) largest: 5.55 cm, smallest: 5.45 cm
 (b) $111.5 \div 5.45 = 20.458$
 so, the maximum is 20
 (c) The shelf could be shorter, e.g. 110.5 cm and the cubes could be wider, e.g. 5.55 cm
 This would mean that only 19 blocks would fit.

Now try these (p 61)

- (a) 205 cm
 (b) 0.34 cm (to 2 d.p.)
 (c) 3 168 750 cm
 (d) 110 cm
- upper bound = $147 - 44.5 - 2.45$
 $= 100.05$ cm
 lower bound = $147 - 45.5 - 2.55$
 $= 98.95$ cm
- (a) height = volume \div cross-sectional area
 upper bound = $320.5 \div 28.5$
 $= 11.25$ cm (to 2 d.p.)
 lower bound = $319.5 \div 29.5$
 $= 10.83$ cm (to 2 d.p.)
 (b) max. volume of drink = 25.5×13.5
 $= 344.25$ cm³
 min. volume of glass = 319.5 cm³
 So no, the drink will not fit.
- upper bound of price to make each cup:
 20.5 ml of lemon juice = $\pounds\left(\frac{20.5}{200}\right) = 10.25$ p
 32.5 g of sugar = $\pounds\left(\frac{32.5}{1000}\right) = 3.25$ p
 total cost to make 1 cup = $10.25 + 3.25$
 $= 13.5$ p
 So, the minimum price Cait can charge to avoid making a loss is 13.5p. There is no upper limit to how much she can charge.

Chapter 1.7 Presenting information

Now try these (p 64)

1.

| | Walk | Cycle | Bus | Total |
|---------|------|-------|-----|-------|
| Year 10 | 40 | 10 | 30 | 80 |
| Year 11 | 20 | 35 | 65 | 120 |
| Total | 60 | 45 | 95 | 200 |

- (a) highest: 15 points
 lowest: 7.5 points
 (b) 46% (to the nearest %)

Now try these (p 67)

- \pounds

51, 53, 57, 59, 61, 63
- (a) 3 (b) 5 (c) 19 (d) 12

Now try these (pp 69)

- Tuesday 13th, Wednesday 14th
 or Thursday 15th
- 12th or 15th August
- (a)

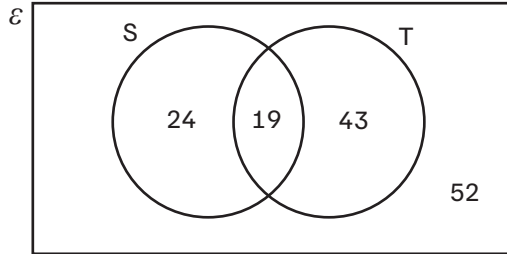
- (b)
-

- (a)

(b) $14 + 3 + 7 = 24$ students

(c) $3 + 7 = 10$ students

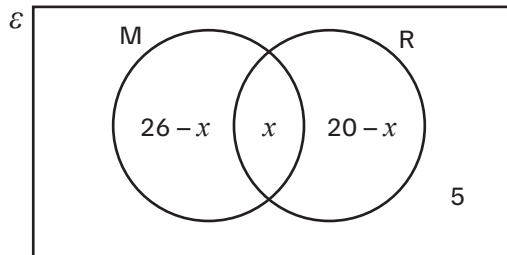
5. (a)



(b) 43 people

(c) 86 people

6.



$$(26 - x) + x + (20 - x) + 5 = 39$$

$$51 - x = 39$$

$$51 - 39 = x$$

$$x = 12$$

(c) $(£37\,000 - £10\,000) \times 0.08 = £2160$

$$\text{Total deductions} = £5940 + £2160 = £8100$$

$$\text{Salary} = £37\,000 - £8100 = £28\,900$$

2. (a) $(£62\,000 - £49\,000) \times 0.42 = £5460$

(b) Amount paid at basic rate:

$$£12\,659.38 - £5460 = £7199.38$$

$$£7199.38 \text{ is } 20\% \text{ of income taxed at basic rate, so } 100\% \text{ is } 5 \times £7199.38 = £35\,996.90$$

So, personal allowance

$$= £49\,000 - £35\,996.90 = £13\,003.10$$

(c) Soshā's salary = £64 500

Amount paid at basic rate:

$$£7199.38$$

Amount paid at higher rate:

$$(£64\,500 - £49\,000) \times 0.42 = £6510$$

$$\text{Total paid} = £7199.38 + £6510$$

$$= £13\,709.38$$

3. (a) $£56\,000 - £12\,000 = £44\,000$

$$£44\,000 \times 0.07 = £3080$$

(b) $£56\,000 - £10\,000 - £3080 = £42\,920$

(c) New income = £61 000

0% is paid on the first £12 000

7% is paid on the next

$$£59\,000 - £12\,000 = £47\,000$$

$$\text{So } £47\,000 \times 0.07 = £3290$$

2% is paid on the next

$$£61\,000 - £59\,000 = £2000$$

$$\text{So } £2000 \times 0.02 = £400$$

So Zichen pays $£3290 + £400 = £3690$

in National Insurance, which is

$$£3690 - £3080 = £510 \text{ more than the previous year.}$$

Chapter 1.8 Money

Now try these (p 71)

1. £12.80

2. £488.40

3. $£578.50 - (13 \times 40) = £58.50$
 $£58.50 \div (13 \times 1.5) = 3$ hours of overtime

Now try these (p 73)

1. (a) $(£37\,000 - £10\,000) \times 0.22 = £5940$

(b) $£37\,000 - £5940 = £31\,060$

Now try these (p75)

- Product A: 70p per 100 ml
Product B: 64p per 100 ml
Product C: 67p per 100 ml
So Product B is the best buy.
- Regular can: 50p per 100 g
Snack can: 43.333p per 100 g
Family can: 43.373p per 100 g
So the snack can is the best value for money.
- Small: £2.75 per 100 ml
Medium: £2.35 per 100 ml
Large: £1.58 per 100 ml
So the large size is the best buy.
- Sample bar: £4.50 per 100 g
Snack bar: £4.00 per 100 g
Regular bar: £2.53 per 100 g
Sharer bar: £2.64 per 100 g
Celebration bar: £2.00 per 100 g
So the celebration bar is the best buy.

Now try these (p 77)

- $£2700 + £450 = £3150$
 $£3150 + £208 = £3358$
 $£3358 - £890 - £105 - £206 - £108 -$
 $£190 = £1859$

Now try these (pp 79–80)

- $710 \times £0.22 = £156.20$
 $£156.20 + £55 = £211.20$
 $£211.20 \times 1.05 = £221.76$
- (a) £400 (c) 1600 kWh
(b) £352
- $(£7.50 \times 4) + (£9 \times 8) = £102$
- (a) 201 units
(b) £413.26
(c) $£1.508 \times 201 = £303.11$

$$(d) \text{ £}14 + \text{£}65 + \text{£}413.26 + \text{£}303.11 \\ = \text{£}795.37$$

Now try these (p 81)

- $£2000 + (5.6\% \text{ of } 2000) \times 3 = £2336$
- $£25\,000 \times 1.078^{10} \times 1.066^{10}$
 $= £100\,392.13$ (to 2 d.p.)
So interest earned = $£100\,392.13 - £25\,000$
 $= £75\,392.13$ (to 2 d.p.)
- After 3 years, Ahmed owes
 $£6594.63 - £2500 = £4094.63$
 $£4094.63 \times 1.032^2 = £4360.88$
- (a) $£500 \times 1.015^4 = £530.68$ (to 2 d.p.)
(b) amount after 6 years:
 $(£500 \times 1.015^6) - £300 = £246.7216\dots$
amount after further 4 years:
 $(£246.7216\dots \times 1.015^4) = £261.8613\dots$
amount if withdrawal hadn't been made:
 $£500 \times 1.015^{10} = £580.2704\dots$
 $£580.2704\dots - £261.8613\dots$
 $= £318.4090\dots = £318.41$ more in the
account when the £300 withdrawal isn't
made. (or £80.27 earned compared to
£238.14 lost)
- Option A:
 $£3000 + (3 \times (£3000 \times 0.035)) = £3315$
Option B:
 $£3000 \times 1.0275^3 = £3254.37$ (to 2 d.p.)
So, option B is cheaper.
- $£9780 \times 1.045^2 \times 1.04^6$
 $= £13\,513.61$ (to 2 d.p.)

Now try these (p 85)

- $AER = \left(1 + \frac{i}{n}\right)^n - 1$
Bank 1: $i = 1.15\% \times 4 = 4.6\% = 0.046, n = 4$
 $AER = \left(1 + \frac{0.046}{4}\right)^4 - 1 = 0.0467\dots = 4.67\dots\%$
Bank 2: $i = 0.52\% \times 12 = 6.24\% = 0.0624,$
 $n = 12$

$$\text{AER} = \left(1 + \frac{0.0624}{12}\right)^{12} - 1 = 0.0642\dots$$

$$= 6.42\%$$

So, bank 2 has the highest AER.

2. (a) $\text{£}1500 \times 1.012^3 = \text{£}1554.65$ (2 d.p.)
 (b) Interest paid = $\text{£}54.65$ (from part (a))

$$\text{£}54.65 + \text{£}50 = \text{£}104.65$$

$$\frac{\text{£}104.65}{\text{£}1500} = 0.06976\dots$$

$$\frac{0.06976\dots}{90} \times 365 = 0.2829\dots$$

$$= 28.29\%$$

(c) $\text{£}1500 \times 1.0001^{12 \times 7}$
 $= \text{£}1512.65$ (2 d.p.)
 Interest paid = $\text{£}12.65$
 $\frac{\text{£}12.65}{\text{£}1500} = 0.008433\dots$
 $\frac{0.008433\dots}{84} \times 365 = 0.03665\dots$
 $= 3.67\%$

This option has a lower APR.

3. (a) $\text{AER} = \left(1 + \frac{i}{n}\right)^n - 1$
 $i = 6.5\% = 0.065, n = 4$
 $\text{AER} = \left(1 + \frac{0.065}{4}\right)^4 - 1 = 0.0666\dots$
 $= 6.67\%$ (to 2 d.p.)

(b) quarterly interest rate = $6.5\% \div 4$
 $= 1.625\%$

$$\text{£}5000 \times 1.016251^4 = \text{£}5333.01$$
 (2 d.p.)

So, amount saved = $\text{£}333.01$ (2 d.p.)

4. $M = \frac{r \times L}{1 - (1 + r)^{-n}}$
 $r = 7.2\% \div 12 = 0.6\% = 0.006$
 $L = \text{£}27\,000 - \text{£}5000 = \text{£}22\,000$
 $n = 6 \times 12 = 72$
 $M = \frac{0.006 \times 22\,000}{1 - (1 + 0.006)^{-72}}$
 $= \text{£}377.19$ (to 2 d.p.)

5. (a) $M = \frac{r \times L}{1 - (1 + r)^{-n}}$

loan A:

$$r = 6.5\% \div 12 = 0.541\dots\% = 0.00541\dots$$

$$L = \text{£}12\,000$$

$$n = 6 \times 12 = 72$$

$$M = \frac{0.00541\dots \times 12\,000}{1 - (1 + 0.00541\dots)^{-72}}$$

$$= \text{£}201.719\dots$$

Total amount paid back after 6 years:

$$\text{£}201.719\dots \times 72 = \text{£}14\,523.77$$
 (to 2 d.p.)

loan B:

$$r = 9\% \div 12 = 0.75\dots\% = 0.0075$$

$$L = \text{£}12\,000$$

$$n = 3 \times 12 = 36$$

$$M = \frac{0.0075 \times 12\,000}{1 - (1 + 0.0075)^{-36}}$$

$$= \text{£}381.5967\dots$$

Total amount paid back after 3 years:

$$\text{£}381.5967\dots \times 36 = \text{£}13\,737.48$$
 (2 d.p.)

So, loan B is cheaper.

(b) Loan A has the lowest monthly payment.

6. (a) $M = P \times r \times \frac{(1 + r)^n}{(1 + r)^n - 1}$
 $P = \text{£}165\,000 - \text{£}16\,500 = \text{£}148\,500$
 $r = 5.5\% \div 12 = 0.4583\dots\% = 0.004583\dots$
 $n = 30 \times 12 = 360$
 $M = 148\,500 \times 0.004583\dots$
 $\times \frac{(1 + 0.004583\dots)^{360}}{(1 + 0.004583\dots)^{360} - 1}$
 $= \text{£}843.17$ (to 2 d.p.)

(b) $\text{£}843.17 \times 360 = \text{£}303\,541.20$

(c) $P = \text{£}148\,500$

$$r = 3.5\% \div 12 = 0.2916\dots\% = 0.002916\dots$$

$$n = 20 \times 12 = 240$$

$$M = 148\,500 \times 0.002916\dots$$

$$\times \frac{(1 + 0.002916\dots)^{240}}{(1 + 0.002916\dots)^{240} - 1}$$

$$= \text{£}618.81$$
 (to 2 d.p.)

$$\text{£}843.17 - \text{£}618.81 = \text{£}224.36$$

(d) Option A: $\text{£}618.81 \times 240 = \text{£}148\,514.40$

$$\text{£}303\,541.20 - \text{£}148\,514.40$$

$$= \text{£}155\,026.80$$

(e) If the variable annual rate remains at 3.5%, the monthly repayment will be the same amount as option A but over an additional 5 years.

So, option B will result in the lowest total amount paid back.

Now try these (p 89)

- (a) $\frac{5}{9}$ (c) $\frac{67}{99}$
(b) $\frac{4}{45}$ (d) $\frac{22}{9}$
- (a) $0.1\dot{6}$ (c) $0.8\dot{3}$
(b) $0.1\dot{4}$ (d) $2.\dot{3}$
- (a) recurring (c) recurring
(b) not recurring (d) not recurring
- Let $x = 0.0\dot{6}$ and $10x = 0.\dot{6}$
 $9x = 0.6$
 $x = \frac{6}{90} = \frac{1}{15}$, and $\frac{1}{15} + \frac{14}{15} = 1$
- $\frac{2}{7} = 0.28571\dot{4}$, so it has 6 recurring digits.

Now try these (p 91)

- (a) Rational because $5 \times 5 = 25$, which is a square number.
(b) Irrational because π is non-repeating and non-terminating.
(c) Rational because it is recurring.
(d) Rational because 2.25 is the result of 1.5×1.5
(e) Rational because $20 \div 5 = 4$, which is a square number.
(f) Irrational because π is non-repeating and non-terminating.

- (a) $3\sqrt{3}$ (g) $4\sqrt{2}$
(b) $6\sqrt{2}$ (h) -6
(c) $4\sqrt{5}$ (i) $\sqrt{5}$
(d) 3 (j) $15\sqrt{2}$
(e) 20 (k) $5\sqrt{3}$
(f) $10\sqrt{2}$ (l) $\frac{23\sqrt{26}}{13}$

- (a) $4\sqrt{6}$ (b) $70\sqrt{2}$ (c) $4\sqrt{5}$

Revision round up (p 92–93)

- 15% of £600 = £90
 $\frac{1}{4}$ of £380 = £95
 $\frac{1}{4}$ of £380 is the larger amount.
- (a) 0.24 (c) 49
(b) 8.84 (d) $\frac{9}{24} = \frac{3}{8}$
- $\frac{13}{50}$ and $\frac{17}{75}$
- $1800 = 1.8 \times 10^3$
 $1.3 = 1.3 \times 10^0$
So, order is:
 2.3×10^{-4} , 9.09×10^{-2} , 1.3, 1800
- $\sqrt[3]{\frac{8.2^2}{8003}} \approx \sqrt[3]{\frac{8^2}{8000}} = \sqrt[3]{\frac{64}{8000}} = \frac{4}{20} = \frac{1}{5}$
- 7200×0.03 and $72 \div \frac{1}{3}$
- $\frac{4}{3}$
- (a) 16 (b) 7
- (a) $\frac{3}{11}$ (c) $\frac{9}{4}$
(b) $\frac{\sqrt{3}}{2}$ (d) $\frac{25}{7}$
- (a) 4.2 (to 2 s.f.) (b) £2.93 (to 2 d.p.)
- $\pounds 20\,000 \times 1.025^3 = \pounds 21\,537.81$ (to 2 d.p.)
- (a) £299 (d) 0.125
(b) 55.5 ml (e) $\frac{20}{24}$ and $\frac{9}{33}$
(c) 100

- 13. (a)** £1010.33
(b) £202.07 (to 2 d.p.)
- 14. (a)** \$995 = £742.54 (to 2 d.p.)
(b) 6.72% increase in the value of the pound (to 2 d.p.)
- 15.** $r = 6\% \div 12 = 0.5\% = 0.005$
 $L = £8750 - £1500 = £7250$
 $n = 3 \times 12 = 36$

$$M = \frac{0.005 \times 7250}{1 - (1 + 0.005)^{-36}}$$
 $= £220.56$ (to 2 d.p.)
- 16.** $1 \frac{1}{15}$
- 17.** maximum tax at basic rate:
 $0.2 \times (£50\,270 - 12\,570) = £7540$
maximum tax at higher rate:
 $0.4 \times (125\,140 - 12\,570) = £29\,948$
 $£7540 + £29\,948 = £37\,488$
So, Jef pays £387 at the additional rate.
 $£387 \div 0.45 = £860$
So, Jef is £860 over the additional rate threshold.
Jef's income = $£125\,140 + £860 = £126\,000$
- 18. (a)** Amount after first three years:
 $£4000 \times 1.025^4 \times 1.04 = £4591.8616\dots$
 $£4591.8616\dots - £200 = £4391.8616\dots$
 $£4591.8616\dots \times 1.04 = £4775.536\dots$
 $= £4775.54$ (2 d.p.)
(b) $(£4775.54 - £4000) \div £4000 = 0.1938\dots$
 $= 19.38\%$ increase (to 2 d.p.)
- 19. (a)** time = distance \div speed
speed = 3.0×10^8 m/s = 3.0×10^5 km/s
 $(1.5 \times 10^8) \div (3.0 \times 10^5) = 500$ s
 $=$ about 8.3 min
(b) distance = $(1.5 \times 10^8) + (5.46 \times 10^7)$
 $= 2.046 \times 10^8$
 $(2.046 \times 10^8) \div (3.0 \times 10^5) = 682$ s
 $= 11.4$ min
- 20. (a)** $17\,024 \div 0.95 = 17920$
 $17\,920 \div 1.12 = 16\,000$ people
(b) $17\,024 - 16\,000 = 1024$
 $1024 \div 1600 = 0.064 = 6.4\%$ increase
- 21. (a)** height = volume \div length \div width
upper bound of height:
 $15 \div 1.445 \div 2.55 = 4.07$ m (to 2 d.p.)
lower bound of height:
 $15 \div 1.455 \div 2.65 = 3.89$ m (to 2 d.p.)
(b) $(2 \times 4.07 \times 1.455) + (2 \times 4.07 \times 2.65) +$
 $(2 \times 1.455 \times 2.65) = 41.1262$ m²
- 22. (a)** 24 builders
(b) that increasing the number of builders will lead to an inversely proportional decrease in the amount of time to finish the development.
- 23.** area of card = 40 cm²
side lengths of card = $\sqrt{40} = 2\sqrt{10}$
area of hole = 10 cm²
side lengths of hole = $\sqrt{10}$
 $4(2\sqrt{10}) + 4(\sqrt{10})$
 $= 8\sqrt{10} + 4\sqrt{10}$
 $= 12\sqrt{10}$ cm
- 24. (a)** $£9200 = £8000 \times 1.03^2 \times x^3$
 $£9200 = £8487.2 \times x^3$
 $1.0839\dots = x^3$
 $x = 1.027\dots$
so, lowest interest rate to earn £1200 after 5 years = 2.8% (round up to 1 d.p.)
(b) $£8000 \times 1.04 \times 1.035 \times 1.03 \times 1.025$
 $\times 1.02 = £9273.10$ (to 2 d.p.)
So yes, this option would generate £1200 in interest.

Section 2: Algebra

Chapter 2.1 Introduction to algebraic expressions

Now try these (p 97)

- | | |
|-------------------|----------------|
| (a) $6xy$ | (f) $6x$ |
| (b) $2x^2$ | (g) $4b$ |
| (c) $\frac{a}{b}$ | (h) $5x^2y^2$ |
| (d) abc | (i) xyz |
| (e) x^3 | (j) hij^2k^2 |
- | | |
|----------------|----------------|
| (a) Expression | (f) Formula |
| (b) Equation | (g) Identity |
| (c) Identity | (h) Equation |
| (d) Expression | (i) Expression |
| (e) Expression | (j) Expression |

Now try these (p 100)

- | |
|-------------|
| (a) $a + 2$ |
| (b) $a - 3$ |
| (c) $b - 5$ |
- | |
|------------------------|
| (a) $\frac{x}{y}$ |
| (b) $\frac{x-5}{y-30}$ |

Now try these (p 101)

- | | |
|----------|--------|
| (a) 28 | (c) 46 |
| (b) 0.25 | (d) 28 |
- 14
- | | |
|-------|-----------|
| (a) 1 | (d) -4 |
| (b) 2 | (e) 1 |
| (c) 4 | (f) -0.25 |
- 2
- | | |
|---------|---------|
| (a) -1 | (e) -15 |
| (b) 108 | (f) 3 |
| (c) 1 | (g) 5 |
| (d) 2 | (h) 0 |
- | | |
|--------|-------|
| (a) 4 | (c) 0 |
| (b) -6 | (d) 1 |
- $-9 + 12.8 = 3.8$

Now try these (p 104)

- | | |
|---------------------|------------------|
| (a) $3x + 15$ | (f) $-x + y$ |
| (b) $8x + 12$ | (g) $-56x - 8y$ |
| (c) $4x + 28$ | (h) $-8x + 4y$ |
| (d) $3x + 12y - 15$ | (i) $3x^2 - 18x$ |
| (e) $10a + 5b - 5c$ | (j) $-2x + 6$ |
- | |
|---------------------------|
| (a) $8x - 16y - 3x + 12y$ |
| $5x - 4y$ |
| (b) $15x + 10y - x + 7y$ |
| $14x + 17y$ |

(c) $28x + 40y - 2x - 3y$

$26x + 37y$

3. $7x^2 + 4x - 6x^2 + 15x - 6$

$x^2 + 19x - 6$

4. (a) $x^2 + 3x - 10$

(e) $8t^2 - 22t + 5$

(b) $a^2 - 6a - 7$

(f) $81b^2 - 1$

(c) $p^2 - 25$

(g) $10f^2 + 23f - 5$

(d) $6st + 10s - 21t - 35$

(h) $8l^2 + 10lm + 2m^2$

5. (a) $4x^2 - 8xy + 4y^2$

(c) $9p^2 - 4q^2$

(b) $25a^2 - 10ab + b^2$

(d) $16s^4 - 4t^2$

6. (a) $x^3 + x^2 - 5x + 3$

(b) $2x^3 + 7x^2 + x - 1$

(c) $5x^3 - 23x^2 - 84x$

(d) $4x^5 - 12x^4 + 4x^3 - 12x^2 + x - 3$

7. (a) $\frac{5x+1}{x^2-1}$

(d) $\frac{2x^2+3x+13}{(x-3)(x+2)}$

(b) $\frac{5x-1}{x^2-1}$

(e) $\frac{x^2+2}{(x+2)(x-4)}$

(c) $\frac{10x+1}{(x+4)(x-3)}$

(f) $\frac{2x^2-5x-54}{(x+6)(x-10)}$

8. (a) $20x - 8$

(b) $9x^2 - 36x$

9. (a) $3x^2 - 6xy$

(b) $2x + 3y$

10. $(2x + 1)(3x + 2) + (x - 3)^2$

$= 6x^2 + 7x + 2 + x^2 - 6x + 9$

$= 7x^2 + x + 11$

(d) $2(6a + 4b + 1)$ (h) $8x^2y(y + 3)$

2. (a) $2(3a + b - 6c)$

(b) $5(9a + 5b - 2c)$

(c) $4b(3a + 1)$

(d) $ab(a - 1)$

(e) $3xy(5x - 4)$

(f) $14x^2y(y + 5)$

(g) $z(x^3y + 1)$

(h) $5x^2(3x + 1)$

Now try these (p 109)

1. (a) $(x + 1)(x + 7)$

(e) $(x - 3)(x - 3)$

(b) $(x + 2)(x + 8)$

(f) $(x - 3)(x - 6)$

(c) $(x + 2)(x + 10)$

(g) $(x - 8)(x + 4)$

(d) $(x - 2)(x + 5)$

(h) $(x - 17)(x + 1)$

2. (a) $(x + 1)(x + 4)$

(e) $2(x - 2)(x - 5)$

(b) $3(x + 1)(x + 1)$

(f) $(3y - 4)(2y - 3)$

(c) $(2x + 1)(x + 2)$

(g) $(3z + 2)(z - 3)$

(d) $(4x + 3)(x + 1)$

(h) $(3t - 2)(2t - 1)$

3. (a) $(x - 1)(x + 1)$

(b) $(x - 10)(x + 10)$

(c) $(x - \sqrt{7})(x + \sqrt{7})$

(d) $(4x - 7)(4x + 7)$

(e) $2(3p - 1)(3p + 1)$

(f) $(2s^2 - 9t^2)(2s^2 + 9t^2)$

(g) $(g^2 - 3\sqrt{2})(g^2 + 3\sqrt{2})$

(h) $2c(3c - 5d)(3c + 5d)$

Now try these (p 106)

1. (a) $2(2a + 3b)$

(e) $6(3a - 2b)$

(b) $4(a + 3b)$

(f) $x(y - x)$

(c) $2(4a + 3b)$

(g) $6x(2x - 1)$

4. (a) $(2x+1)(x-2)$ (e) $(3y-1)(4y-1)$
 (b) $(3x+2)(x+4)$ (f) $(4y-7)(2y+3)$
 (c) $(2x-1)(4x-1)$ (g) $(4-3y)(2y-3)$
 (d) $(6x-1)(2x+5)$ (h) $(-7y-2)(y-5)$

Now try these (p 111)

1. (a) $4x^4$ (e) xy^2
 (b) $12x^{10}$ (f) $\frac{37y}{13x}$
 (c) y^4 (g) $2x^2 + 2x$
 (d) $3x^3$ (h) $\frac{4p^5}{q}$
2. (a) $(x^2+2)(x^2+3) = 0$
 (b) $(2\sqrt{x}+1)(3\sqrt{x}-4) = 0$
 (c) $3c^3d^4(4c+3d)(4c-3d)$
 (d) $(\sqrt{x}-2)(\sqrt{x}-5) = 0$
 (e) $(x^{\frac{1}{3}}+1)(3x^{\frac{1}{3}}-5) = 0$
 (f) $(2^x-1)(2^x-8) = 0$
3. (a) $\frac{3}{z}$ (h) $4x$
 (b) $4x$ (i) $2(2x-1) = 4x-2$
 (c) $3xy^2$ (j) $\frac{x-2}{x+4}$
 (d) $3z^2$ (k) $\frac{2x+3}{3} = \frac{2x}{3} + 1$
 (e) $\frac{7c}{2ad}$ (l) $\frac{a-2}{b}$
 (f) $\frac{5y^2z}{x}$ (m) $\frac{x^2-3}{x^2+2}$
 (g) $\frac{5x-6}{2x-3}$ (n) $\frac{3(x+2)}{x+5}$

Now try these (p 113)

1. (a) $x = \frac{y-6}{3}$ (d) $x = \frac{y-1}{3}$
 (b) $x = \sqrt{\frac{y-1}{2}}$ (e) $x = \sqrt{10y-5}$
 (c) $x = 2y-3$ (f) $x = \sqrt{\frac{y}{3}-7}$

2. (a) $a = \frac{c+4}{c-4}$ (c) $a = \sqrt{-4c-3}$
 (b) $a = \frac{3b+9}{4b+9}$ (d) $a = \frac{4bc+7}{bc-1}$
3. (a) $t = \frac{2s}{u+v}$
 (b) $u = \frac{2s}{t} - v$
 (c) $v = \frac{2s}{t} - u$
4. (a) $a = \frac{v^2-u^2}{2s}$
 (b) $s = \frac{v^2-u^2}{2a}$
 (c) $u = \sqrt{v^2-2as}$
5. $\cos A = \frac{b^2+c^2-a^2}{2bc}$
6. (a) $x = \frac{y}{z^4}$ (d) $x = \frac{y^2}{2z}$
 (b) $x = \sqrt[4]{\frac{y}{z^2}}$ (e) $x = yz^2$
 (c) $x = \frac{y}{z^3}$ (f) $x = \frac{2wy}{z^2}$
7. $r = \sqrt{\frac{A}{\pi}}$
8. (a) $j = \frac{i^2-hi}{h-1}$ (d) $t = \frac{-6r}{r-s}$
 (b) $p = \frac{q+rs^2}{s^2-1}$ (e) $d = \frac{9g^3}{3-9g^2}$
 (c) $g = \sqrt{\frac{-6}{h-9}}$ (f) $h = \frac{A}{2\pi r} - r$

Chapter 2.2 Solving equations and inequalities

Now try these (p 117)

- | | |
|---------------|---------------|
| (a) $x = 4$ | (f) $x = 35$ |
| (b) $x = 4$ | (g) $x = 21$ |
| (c) $x = -4$ | (h) $x = 0.6$ |
| (d) $x = 3.5$ | (i) $x = 5.5$ |
| (e) $x = 0.5$ | (j) $x = 0.5$ |
- | | |
|-----------------------|-----------------------|
| (a) $x = \frac{5}{3}$ | (e) $x = 6$ |
| (b) $x = 4$ | (f) $x = \frac{5}{3}$ |
| (c) $x = 15$ | (g) $x = 40$ |
| (d) $x = 31$ | (h) $x = 7$ |
- | | |
|--------------|--------------|
| (a) $x = 5$ | (c) $x = 13$ |
| (b) $x = -2$ | (d) $x = 3$ |
- | | |
|---------------|---------------|
| (a) $x = 100$ | (c) $x = -2$ |
| (b) $x = -2$ | (d) $x = -12$ |
- let $x =$ width and $(x + 5) =$ length
perimeter $= 2x + 2(x + 5) = 38$
 $4x + 10 = 38$
 $4x = 28$
 $x = 7$ m
So, width $= 7$ m and length $= 12$ m

Now try these (pp 118–119)

- | |
|---------------------|
| (a) $3x - 6 = 15$ |
| $3x = 21$ |
| $x = 7$ |
| (b) $5x + 30 = 35$ |
| $5x = 5$ |
| $x = 1$ |
| (c) $5x + 35 = 35$ |
| $5x = 0$ |
| $x = 0$ |
| (d) $9x - 12 = 15$ |
| $9x = 27$ |
| $x = 3$ |
| (e) $20x - 5 = 35$ |
| $20x = 40$ |
| $x = 2$ |
| (f) $20x + 5 = -35$ |
| $20x = -40$ |
| $x = -2$ |
- $5x - 15 - 3x - 3 = 0$
 $2x = 17$
 $x = \frac{17}{2}$
- $2x - 8 - \frac{7}{2}x - 21 = 4$
 $-\frac{3}{2}x = 33$
 $x = -22$
- | |
|--|
| (a) When expanding the brackets, Ceri multiplied -3 by -3 and got -9 , not 9 |
| (b) $x = -6$ |
- $A = \frac{1}{2}bh$

$$36 = \frac{1}{2} \times (x - 5) \times 6$$

$$36 = 3x - 15$$

$$51 = 3x$$

$$x = 17$$

(g) -4, -3, -2, -1, 0, 1, (i) 0, 1, 2, 3, 4, 5
2, 3, 4

(h) 3, 4, 5, 6, 7, 8, 9 (j) 12, 13, 14, 15,
16, 17, 18

2. $-1 \leq x < 4$

-1, 0, 1, 2, 3

3. $7.5 \leq n \leq 10.5$

8, 9, 10

4. $5.666... < n \leq 10$

6, 7, 8, 9, 10

5. (a) $3 < x \leq 7$

(b) $-\frac{2}{3} < x < 3$

(c) $\frac{1}{2} \leq x < 4$

6. $80 + 0.5m \leq 350$

$0.5m \leq 270$

$m \leq 540$ miles

7. $4x + 25 < 7x + 4$ so $x > 7$

$7x + 4 \leq 11x - 20$ so $x \geq 6$

So $x > 7$

Now try these (p 120)

1. (a) $a < 10$ (c) $cd < 8.25$

(b) $b \geq 7.5$ (d) $\frac{e}{f} \leq 4$

2. (a) $p \geq 10$

$p \leq 20$

(b) $10 \leq p \leq 20$

Now try these (p 122)

1. (a) $3x > 9$ (g) $x - 10 < 28$

$x > 3$ $x < 38$

(b) $x - 2 > 2$ (h) $2x \leq 9$

$x > 4x$ $x \leq 4.5$

(c) $x - 7 < 3$ (i) $x - 5 \geq 0$

$x < 10$ $x \geq 5$

(d) $x + 5 \leq -21$ (j) $-2x \leq 0$

$x \leq -26$ $x \geq 0$

(e) $-3x < -12$ (k) $x - 21 \geq 0$

$x > 4$ $x \geq 21$

(f) $2x \geq 28$ (l) $x + 5 < 180$

$x \geq 14$ $x < 175$

Now try these (p 125)

1. (a) 3, 4, 5, 6, 7, 8, 9, 10 (d) 16, 17, 18

(b) -4, -3, -2, -1, 0 (e) -2, -1, 0, 1, 2

(c) 1, 2, 3, 4, 5, 6, 7 (f) 16, 17, 18, 19

Now try these (p 126)

1. (a) $x = 2, x = 3$ (g) $x = -6, x = 4$

(b) $x = 3$ (h) No solutions

(c) $x = 5$ (i) $x = \frac{2}{3}, x = 1$

(d) $x = -6, x = 6$ (j) $x = -\frac{4}{3}, x = 3$

(e) $x = -1, x = -2$ (k) $x = 0, x = 1$

(f) $x = -7$ (l) $x = -2, x = -0.2$

2. (a) $x = 2, x = 4$ (c) $x = 0, x = 10$

(b) $x = -4, x = 3$ (d) $x = 0$

Now try these (p 127)

1. (a) $x = 2, x = -1.5$ (e) No solutions
(b) $x = -6.69,$
 $x = 1.19$ (f) $x = -1.18,$
 $x = 10.18$
(c) No solutions (g) $x = 0$
(d) $x = -10.60,$
 $x = 1.60$ (h) $x = -1.24, x = 0,$
 $x = 0.91$

Now try these (p 129)

1. (a) $x = 2, y = 3$
(b) $x = 2, y = 4$
(c) $x = 1, y = 2$
(d) $x = 4, y = -1$
(e) $x = -3, y = -2.5$
2. $3a + 2c = 61.5$
 $2a + 3c = 65$
 $a = 10.9, c = 14.4$
So an adult ticket costs £10.90, and a child ticket costs £14.40.

Now try these (p 131)

1. $x = 1$ gives $x^3 - x - 2 = -2$
 $x = 2$ gives $x^3 - x - 2 = 4$
So a solution exists between 1 and 2.
2. $x = 0$ gives $9x^3 - 9x + 1 = 1$
 $x = 0.2$ gives $9x^3 - 9x + 1 = -0.728$
So a solution exists between 0 and 0.2.
3. $x = 5$
4. $x = 1.6$
5. $x = 0.34$

Now try these (p 133)

1. (a) $x = \frac{7}{5}$ (d) $x = \frac{3}{8}$
(b) $x = \frac{15}{8}$ (e) $x = \frac{17}{6}$
(c) $x = -\frac{8}{35}$ (f) $x = -\frac{18}{5}$
2. (a) $x = 10$ (c) $x = 5$
(b) $x = 4$ (d) $x = 5$
3. (a) $x = 2$ (d) $x = 5$
(b) $x = -0.653,$
 $x = 7.653$ (e) $x = -0.317,$
 $x = 6.317$
(c) $x = -5.333,$ (f) $x = -10, x = 1$
 $x = 0$
4. $\frac{64}{x} = \frac{96}{x+2}$
 $x = 4$, so Set A contains 4 numbers and Set B contains 6 numbers.
5. $\frac{3}{x-7} = \frac{x+3}{2x-1}$
 $x = 5 + \sqrt{43} = 11.55743\dots$
Note there is also a negative solution to this equation, but this would imply a negative density and volume so is not correct.
6. $\frac{x}{x+3} = \frac{2x+7}{3x-4}$
 $x = 18.15660\dots$
Note there is also a negative solution to this equation, but this would imply a negative time and speed so is not correct.

Chapter 2.3

Sequences

Now try these (p 136)

- (a) 9, 11, 13, 15, 17

(b) 207

(c) $107 + 307 = 414$
- (a) 3, 5, 7, 9, 11

(b) No. $2n + 1$ is always an odd number, but 250 is even

Now try these (p 138)

- (a) Subtract 3 from the previous term to find the next term.

(b) $-3n + 16$

(c) $-3(100) + 16 = -300 + 16 = -284$
- (a) $3n - 2$

(b) $19 - 4n$

(c) $23n + 77$

(d) $-6n - 6$
- (a) $4n - 10$

(b) $80 - 10 = 70$

(c) All terms in the sequence are even so Jerome is correct, although his reasoning is incorrect. All terms in the sequence are even, regardless of whether n is even.

(d) No. Every term in the sequence is formed by multiplying n by 4 (even) and subtracting 10 (even), so every term will be even.
- (a) 18, 16, 14

(b) $20 - 2n = 8$
 $12 = 2n$
 $n = 6$

(c) $20 - 2n < 0$
 $20 < 2n$
 $n > 10$

So, the first negative value in the sequence is the 11th term, which is
 $20 - 2(11) = 20 - 22 = -2$

5. $2n + 4$

Now try these (p 139)

- (a) 39, 60, 85, 114

(b) 35, 45, 57, 71

(c) 86, 112, 140, 170

(d) 45, 60, 77, 96

(e) 70, 102, 140, 184
- (a) $n^2 + n - 1$

(b) $n^2 + 5n - 2$

(c) $n^2 - n + 19$

(d) $-n^2 - 1$

(e) $-1.5n^2 + 1.5n$
- (a) $4n^2 + n - 1$

(b) $4n^2 + n - 1 > 100$
 $4n^2 + n - 101 > 0$
using the quadratic formula:
 $n > 4.9...$
So, the first term that is greater than 100 is the 5th term, which is 104

(c) $4(107)^2 + 107 - 1 = 45\,902$
- (a) 38, 32, 22, 8, -10, -32, -58, -88, -122, -160

(b) $40 - 2n^2 < -12$
 $-2n^2 < -52$
 $n > \sqrt{26} = 5.099...$
So, the first value in the sequence that is less than -12 is the 6th term, which is -32

(c) Yes, because anything multiplied by 2 is always even, so $40 - 2n^2$ is always even. The only even prime number is 2, which is not in the sequence.

5. (a) 5, 19, 41, 71, 109

(b) $4n^2 + 2n - 1 > 100$

$$4n^2 + 2n - 101 > 0$$

using the quadratic formula:

$$n > 4.78\dots$$

So, the first value in the sequence that is greater than 100 is the 5th term, which is 109

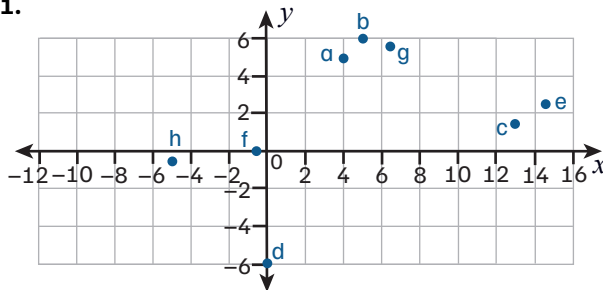
(c) Yes, because anything multiplied by 2 or 4 is always even, so $4n^2 + 2n - 1$ will always be odd.

6. n^2

Chapter 2.4 Coordinates and graphs

Now try these (pp 140–141)

1.



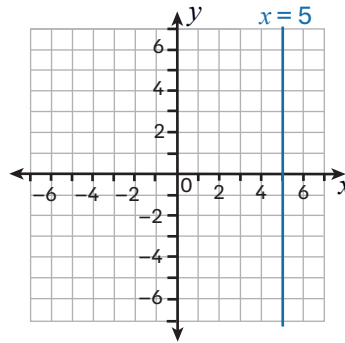
2. (a) $P = (6, 4)$

(b) $Q = (2, -4)$

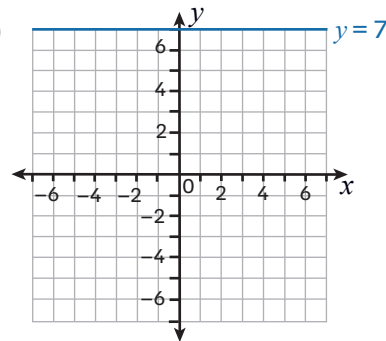
(c) $R = (-10, 0)$

Now try these (p 142)

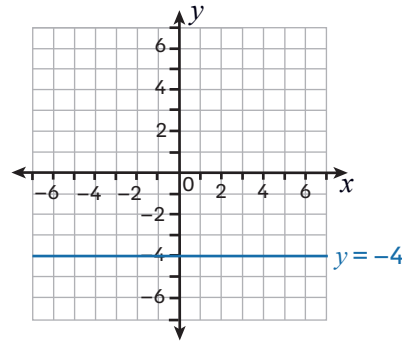
1. (a)



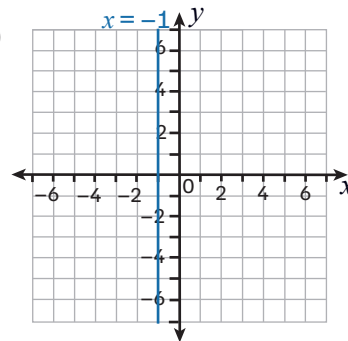
(b)

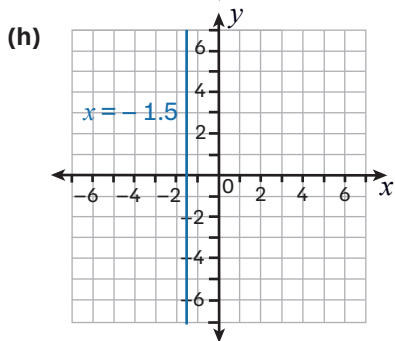
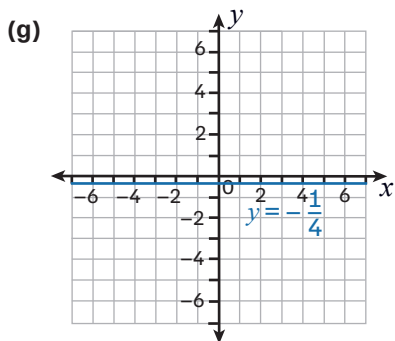
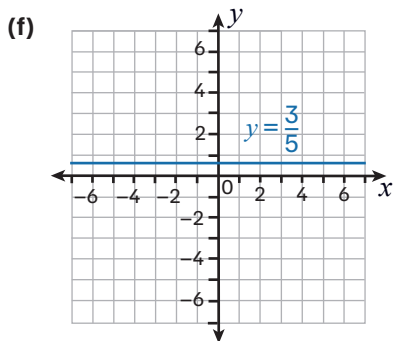
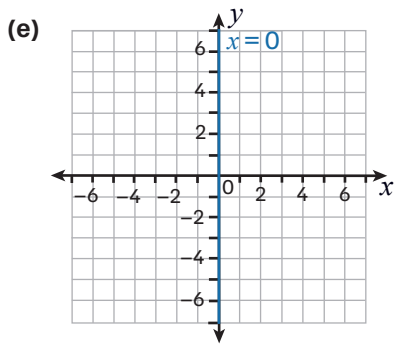


(c)



(d)





2. A $x = 4$
 B $y = 2$
 C $x = -3$
 D $x = 6.5$
 E $y = -4.5$

Now try these (p 143)

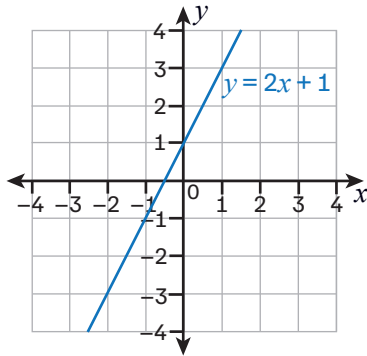
1. (a) Yes (e) No
 (b) No (f) Yes
 (c) No (g) Yes
 (d) Yes (h) Yes
2. (a) $m = 3$ $c = 1$
 (b) $m = -2$ $c = 2$
 (c) $m = \frac{1}{2}$ $c = 6$
 (d) $m = 2$ $c = 4$
 (e) $m = 1$ $c = -4$
 (f) $m = -1$ $c = 5$
 (g) $m = 1$ $c = 6$
 (h) $m = \frac{2}{5}$ $c = -2$

Now try these (p 147)

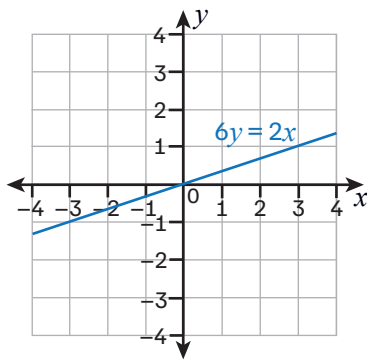
1. (a) 1
 (b) -2
 (c) -2
2. C
3. B
4. A

Now try these (p 149)

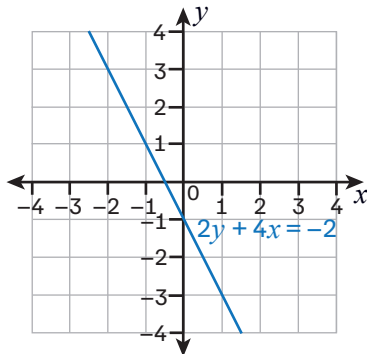
1. (a)



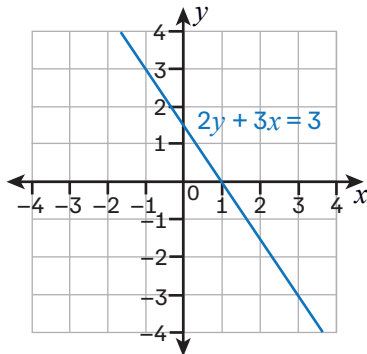
(b)



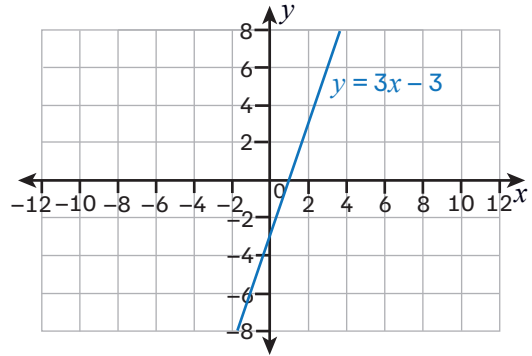
(c)



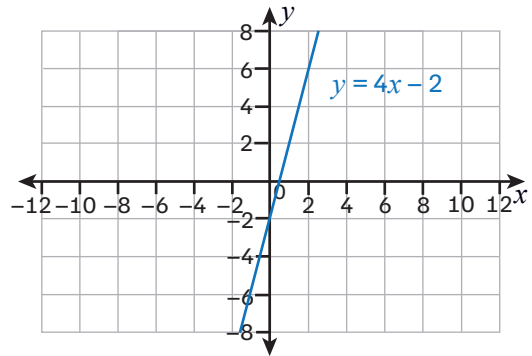
(d)



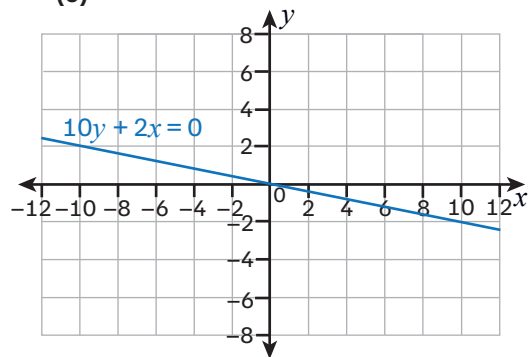
2. (a)



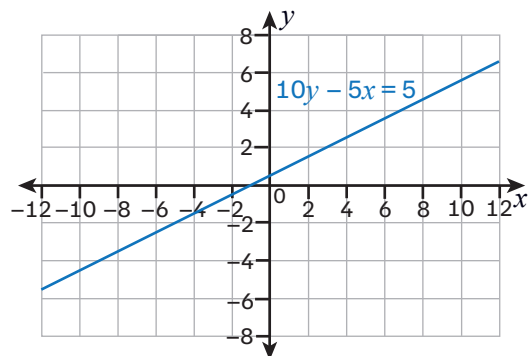
(b)



(c)

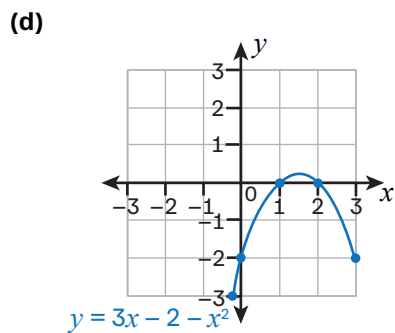
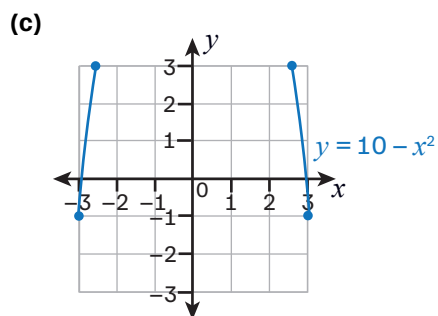
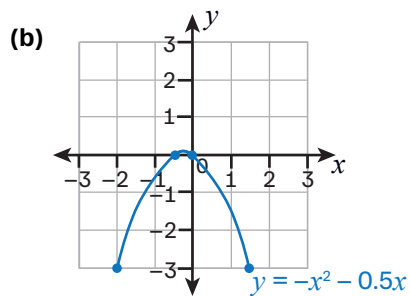
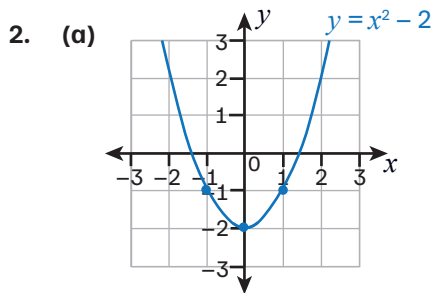


(d)

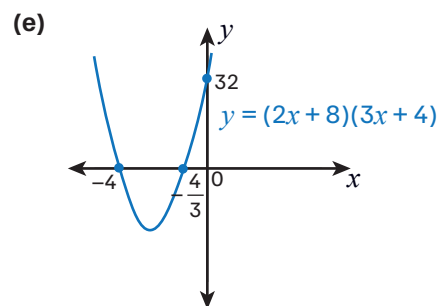
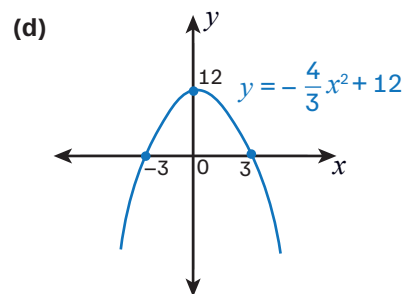
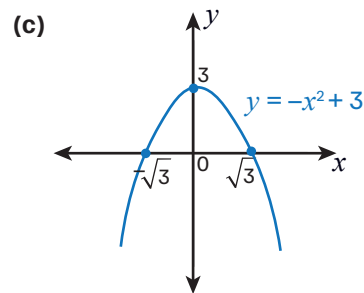
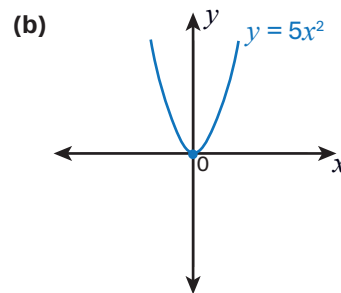
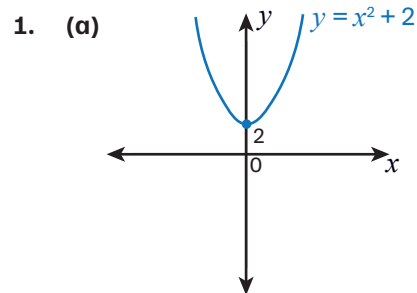


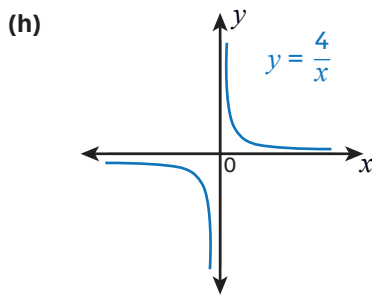
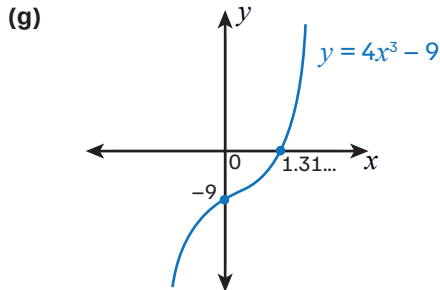
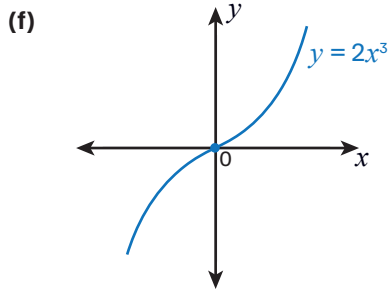
Now try these (p 150)

1. (a) \cup (c) \cap
 (b) \cap (d) \cap



Now try these (p 152)





2. A: $y = x^2 - 2$
 B: $y = -x^3$
 C: $y = x^3$
 D: $y = 2 - x^2$

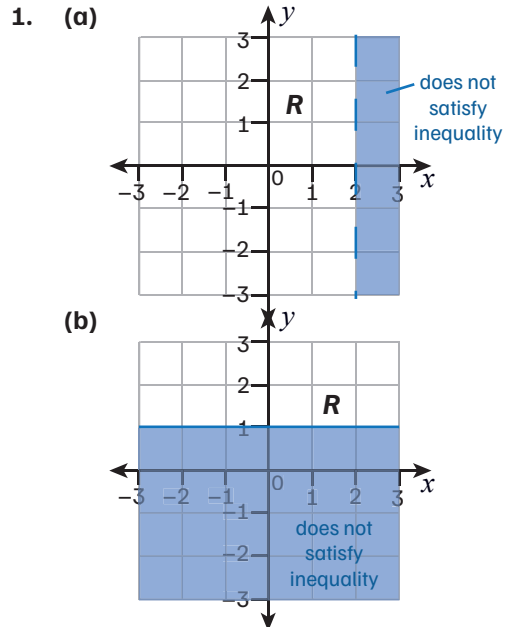
Now try these (p 153)

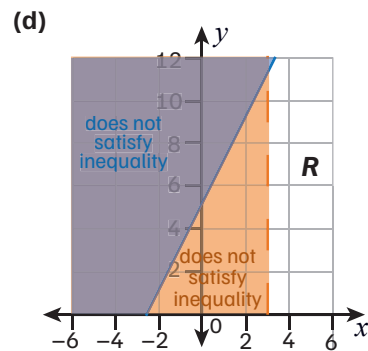
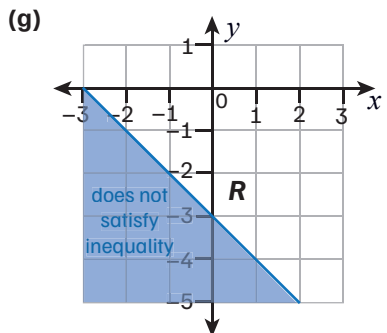
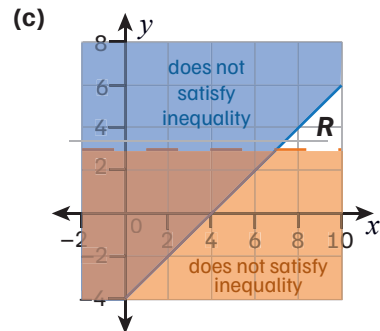
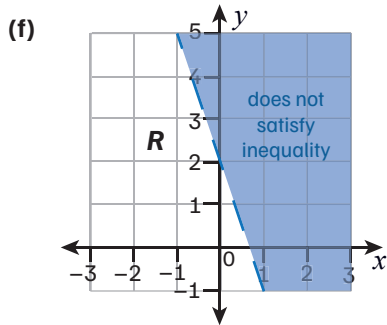
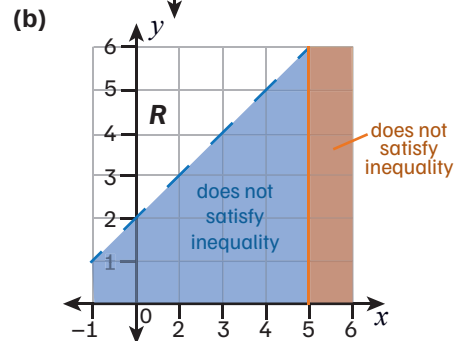
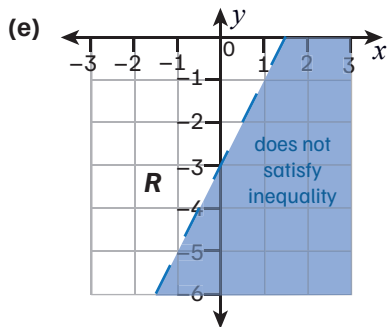
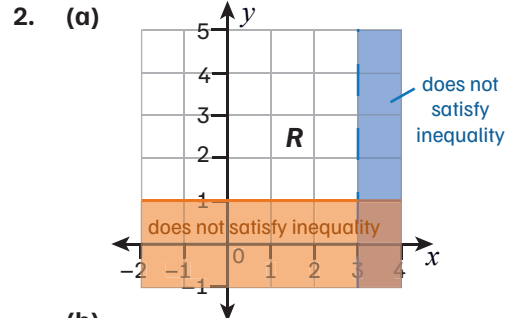
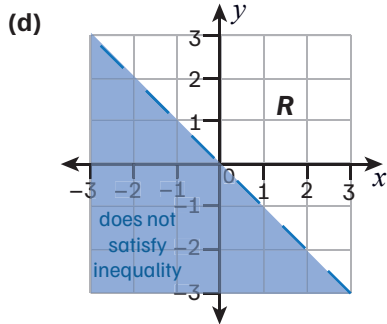
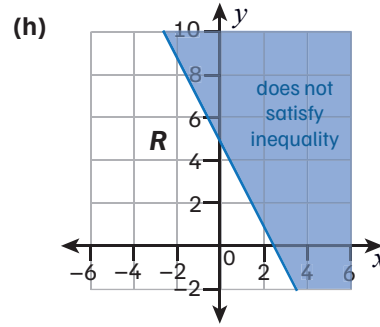
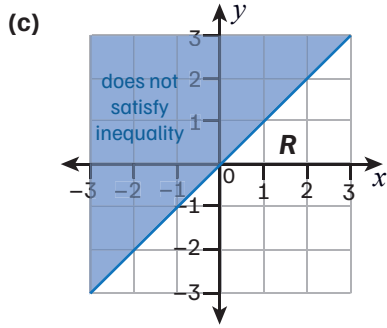
1. (a) (0, 1.0)
 (b) (0.3, 0.3)
 (c) (-2.5, -0.5)
 (d) no intersection
 (e) (2.0, 16.0)
 (f) (1.0, 2.5)

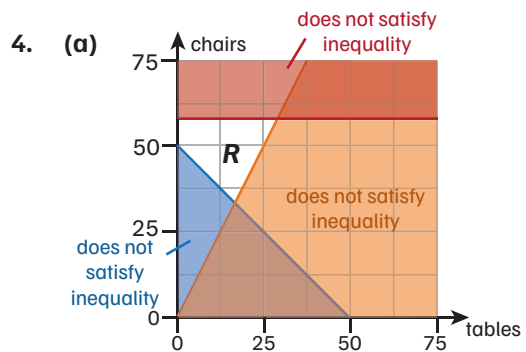
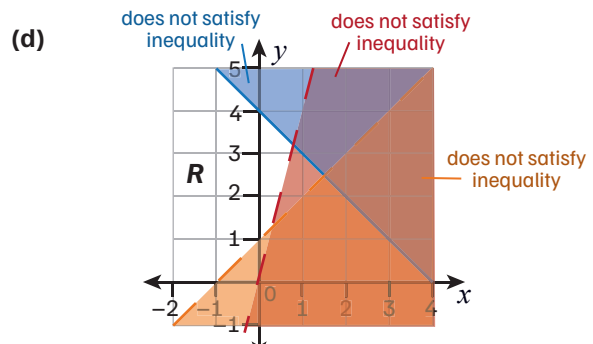
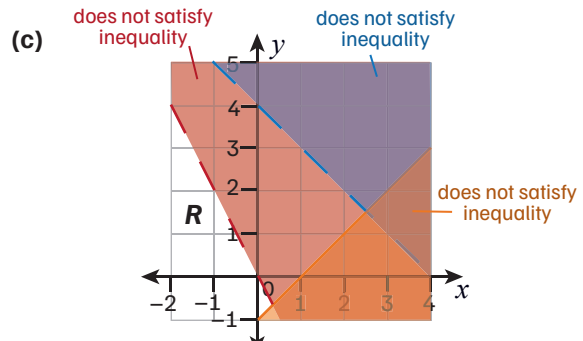
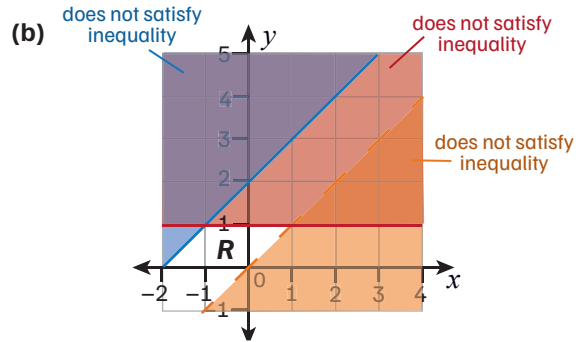
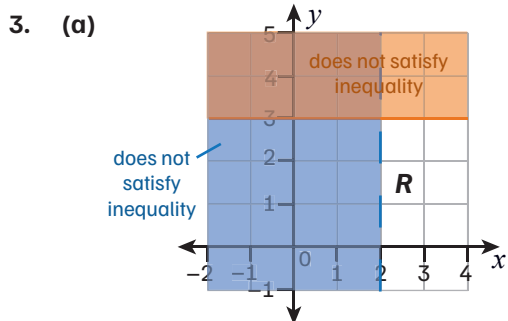
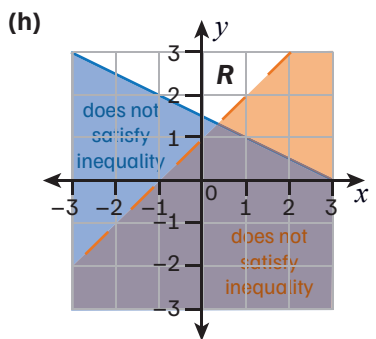
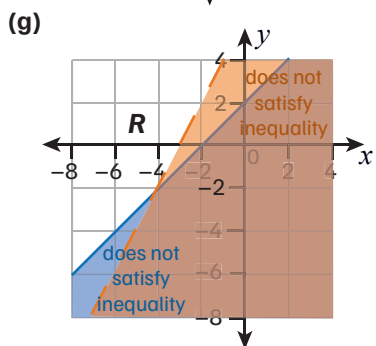
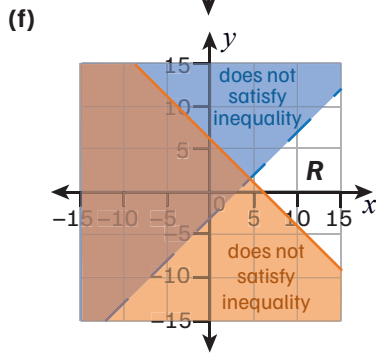
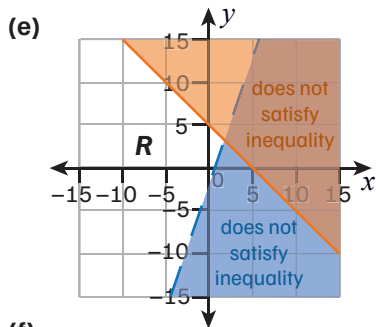
Now try these (p 155)

1. (a) no intersection
 (b) (-7.1, 0.5) and (0.1, 0.5)
 (c) no intersection
 (d) (0, -15.0)
 (e) (-1.5, 0) and (1.0, 5.0)
 (f) no intersection
 (g) (-1.0, 2.0) and (-0.3, 2.7)
2. (a) $x = -3$ and $x = 2$
 (b) $x = -4$ and $x = 3$
 (c) $x = 1$ and $x = 4$
 (d) $x = -1.7$, $x = -0.6$ and $x = 0.9$
 (e) $x = -2.7$ and $x = 2.2$
 (f) $x = 1.7$
 (g) $x = -0.7$

Now try these (p 157)







(b) smallest = 0, largest = 30

(c) 34 chairs

Chapter 2.5 Real-life graphs

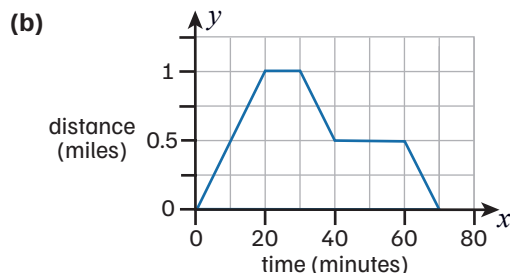
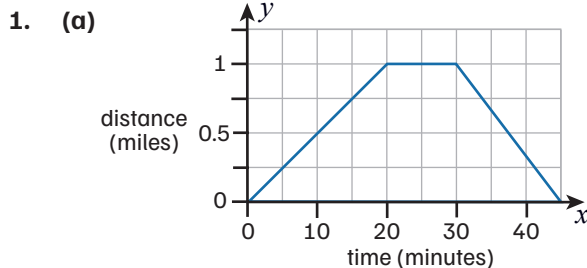
Now try these (pp 159–160)

- (a) £13
(b) €9
- (a) 16 inches
(b) 100 centimetres

Now try these (p 161)

- (a) £25 per hour
(b) £25
(c) £275
- (a) approximately £52.50
(b) £0.50
(c) $y = 0.5x + 35$

Now try these (p 163)



- (a) A and C
(b) 8 m/s
(c) 2 m/s

Now try these (p 165)

- (a) $\text{area} = \frac{1}{2}(1.1 + 2.8)5 + \frac{1}{2}(2.8 + 3.75)5$
 $+ \frac{1}{2}(3.75 + 1.5)5 + \frac{1}{2}(1.5 + 0)5$
 $= 43 \text{ m}$
 (b) $\text{area} = \frac{1}{2}(1.1 + 3.75)10 + \frac{1}{2}(3.75 + 0)10$
 $= 43 \text{ m}$

So, there is no difference in the area under the graph when using four strips compared to when using two.

- (a) $\text{area} = \frac{1}{2}(0 + 1)1 + \frac{1}{2}(1 + 4)1 +$
 $\frac{1}{2}(4 + 9)1 + \frac{1}{2}(9 + 16)1$
 $= 22$
 (b) The answer is an overestimate because the curve is concave, so each trapezium is slightly larger than the corresponding area under the curve.

- The graph shows the rate of flow of water out of the tank, so the area under the entire graph is equal to the total amount of water removed from the tank.

using three strips:

$$\begin{aligned} \text{area} &= \frac{1}{2}(2 + 1.3)20 + \frac{1}{2}(1.3 + 0.5)20 \\ &+ \frac{1}{2}(0.5 + 2)20 \\ &= 76 \text{ litres} \end{aligned}$$

Now try these (p 167)

- (a) Using a tangent passing through the points (0.25, 0.6) and (1, 1.8):
 speed = 1.6 m/s
 (b) Speed = 0 m/s, so the ball is stationary (i.e. it has stopped rising upwards and is about to begin falling towards the ground).

2. (a) The car was travelling fastest between 20 and 25 seconds.

Accept answers within this time frame.

- (b) Using a tangent at $t = 22.5$ seconds passing through the points (20, 19) and (25, 32):

$$\text{acceleration} = 2.6 \text{ m/s}^2$$

- (c) Using three strips:

$$\begin{aligned} \text{area} &= \frac{1}{2}(0 + 7)10 + \frac{1}{2}(7 + 19)10 \\ &\quad + \frac{1}{2}(19 + 39)10 \\ &= 455 \text{ m} \end{aligned}$$

Revision round up (p 168)

1. $b = -2$

2. $\frac{x+3}{y} = 3$, as it can be rearranged to be in the form $y = mx + c$ ($y = \frac{x}{3} + 1$)

3. (a) $x = 5$

(b) $x = \sqrt{\frac{y-4}{-3}}$

(c) $3(2x^2y - 6x + 1)$

4. $y \leq -\frac{1}{2}x + 5$ and $y > x - 5$

5. $(2312 \div 14 =) 165$ remainder 2 or $165\frac{2}{14}$
or (n th term) $2312 - 14n$ or equivalent
 $2312 - 14 \times 165 (= 2)$ or $2312 - 14 \times 166 (= -12)$
(First number in the sequence < 0 is) -12
 166 (th term)

6. $\frac{x+2}{x+5}$

7. $x(x+7) = 120 \text{ m}^2$
 $x^2 + 7x - 120 = 0$
 $(x+15)(x-8) = 0$
 $x = 8$ so height = 8 m and length = 15 m

8. 1 coat costs £55, 1 jumper costs £30
2 coats and 1 jumper costs £140

9. (a) $c = a - 1$

$$3c + 2a = 12$$

(b) $c = \text{£}2, a = \text{£}3$

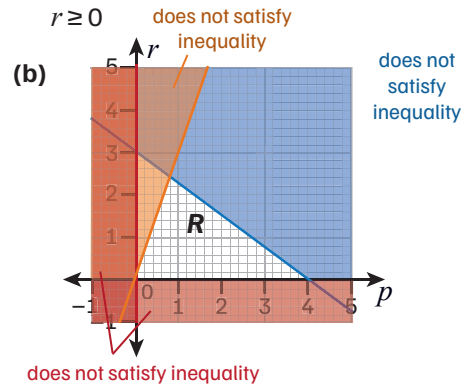
10. 20 m

11. (a) $3p + 4r \leq 12$

$$r \leq 3p$$

$$p \geq 0$$

$$r \geq 0$$



(c) £1.50

12. $n(n+1) = 90$

$$n = 9$$

So, the numbers are 9 and 10

13. (a) (0, -3)

(b) $\frac{4}{3}$

(c) 4

14. $x = -2.70$ and $x = 0.37$

15. (a) $4(1)^3 - 2(1) - 5 = -3$

$$4(2)^3 - 2(2) - 5 = 23$$

(b) $x = 1.2$

16. (a) Using a tangent passing through points (40, 65) and (50, 75):

$$\text{acceleration} = 1 \text{ m/s}^2$$

(b) 3367.5 m

$$\begin{aligned}
 17. \quad & \frac{4x^2 - 9}{6x + 9} \div \frac{2x^2 - 3x}{x} \\
 & \frac{(2x + 3)(2x - 3)}{3(2x + 3)} \div \frac{x(2x - 3)}{x} \\
 & \frac{(2x - 3)}{3} \div (2x - 3) \\
 & \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & (2x + 1)^2 + (2x + 3)^2 \\
 & = 4x^2 + 4x + 1 + 4x^2 + 12x + 9 \\
 & = 8x^2 + 16x + 10 \\
 & = 8(x^2 + 2x + 1) + 2
 \end{aligned}$$

$$\begin{aligned}
 19. \quad (a) \quad & (x + 1)(2x) + (3x - 1)(3x + 1) = 47 \\
 & 2x^2 + 2x + 9x^2 - 1 = 47 \\
 & 11x^2 + 2x - 1 = 47 \\
 & 11x^2 + 2x - 48 = 0
 \end{aligned}$$

$$(b) \quad x = 2$$

$$\begin{aligned}
 20. \quad \text{surface area} = & 2(x)(x - 4) + 2(x)\left(\frac{1}{2}x\right) + 2(x - 4)\left(\frac{1}{2}x\right) = 90 \\
 & 4x^2 - 12x = 90 \\
 & 2x^2 - 6x - 45 = 0
 \end{aligned}$$

$$x = 6.47 \text{ (to 3 s.f.)}$$

$$\begin{aligned}
 21. \quad & 3c - 7.2 = 2t \text{ so } 9c - 21.6 = 6t \text{ (1)} \\
 & 5c - 12.7 = 3t \text{ so } 10c - 25.4 = 6t \text{ (2)} \\
 & (2) - (1) \\
 & c - 3.8 = 0 \\
 & c = \text{£}3.80 \text{ so } t = \text{£}2.10
 \end{aligned}$$

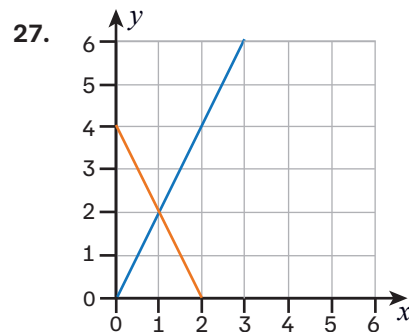
$$\begin{aligned}
 22. \quad & \frac{1}{2}(x + 1)(x + 2) = (x + 3)(x - 2) \\
 & \frac{1}{2}(x^2 + 3x + 2) = x^2 + x - 6 \\
 & x^2 + 3x + 2 = 2x^2 + 2x - 12 \\
 & x^2 - x - 14 = 0 \\
 & x = 4.275 \text{ cm (to 3 d.p.)}
 \end{aligned}$$

$$23. \quad (-4, 2) \text{ and } (1, 12)$$

$$24. \quad (0, -2) \text{ and } (2, 0)$$

$$\begin{aligned}
 25. \quad & (w + 5)^2 + w^2 = 22^2 \\
 & w^2 + 10w + 25 + w^2 = 484 \\
 & 2w^2 + 10w - 459 = 0 \\
 & w = 12.85 \text{ cm, so } l = 17.85 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & 2s + 5l = 250 \text{ so } 6s + 15l = 750 \text{ (1)} \\
 & 6s + 3l = 270 \text{ (2)} \\
 & (1) - (2) \\
 & 12l = 480 \\
 & l = 40, \text{ so } s = 25
 \end{aligned}$$



small bag = £1

large bag = £2

Section 3: Geometry and Measures

Chapter 3.1 Properties of angles and shapes

Now try these (pp 174–175)

- (a) rhombus
(b) regular hexagon
(c) isosceles triangle
(d) rectangle or parallelogram
(e) regular pentagon
(f) equilateral triangle
- rectangle: 2 pairs
square: 4 equal, 4 total; all 90°
rhombus: 2 pairs
parallelogram: 2 equal pairs, 4 total; 2 equal pairs; 2 pairs
isosceles triangle: 2 equal, 3 total; none

Now try these (p 176)

- (a) $\frac{360^\circ}{5} = 72^\circ$
(b) $180^\circ - 72^\circ = 108^\circ$
(c) $5 \times 108^\circ = 540^\circ$
- $180^\circ - 162^\circ = 18^\circ$ (interior angle)
 $\frac{360^\circ}{18^\circ} = 20$ sides
- (a) $\angle PQR = 108^\circ$
 $x = (180^\circ - 108^\circ) \div 2 = 72^\circ \div 2 = 36^\circ$
(angles in triangle add to 180° , triangle is isosceles)

(b) trapezium

(c) $y = 108^\circ - 36^\circ = 72^\circ$

Now try these (p 177)

- (a) circumference
(b) diameter
(c) radius
(d) chord
- A: (minor) sector
B: (minor) segment

Now try these (p 179)

- cube: 12 edges; 8 vertices
cuboid: 6 faces; 12 edges; 8 vertices
sphere: 1 face, 0 edges
square-based pyramid: 5 faces; 5 vertices
cone: 2 faces; 1 edge; 1 vertex
- (a) (from counting) 7 vertices
(b) (from counting) 7 faces
(c) $7 + 7 = 2 + E$
 $E = 14 - 2 = 12$
(d) 8 faces, 12 vertices, 18 edges

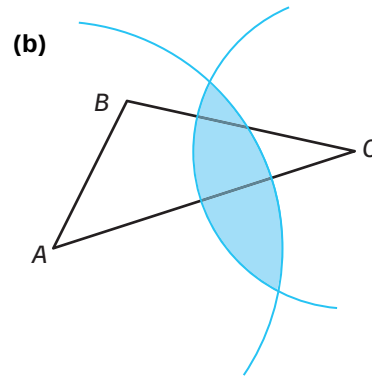
Chapter 3.2 Using mathematical equipment

Now try these (p 184)

- Check constructions using a protractor.

2. Check construction using a ruler.
3. Check construction using a ruler.
4. Check construction using a ruler and a protractor.
5. (a) Check construction using a ruler and a protractor. The 80° angle should be at vertex Z.

(b) Check construction using a ruler and a protractor. The 100° angle should be at vertex Z.
6. Check constructions using a ruler and a protractor. The angles either side of the bisector should both equal 90° .
7. Check constructions using a protractor. The angles either side of the bisector should be the same.

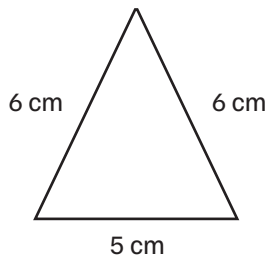


Chapter 3.3 Maps, scales, bearings and 2-D drawings

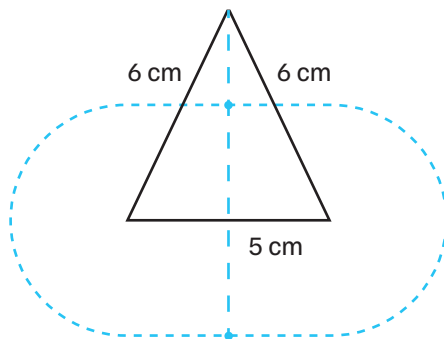
Now try these (p 187)

Now try these (p 185)

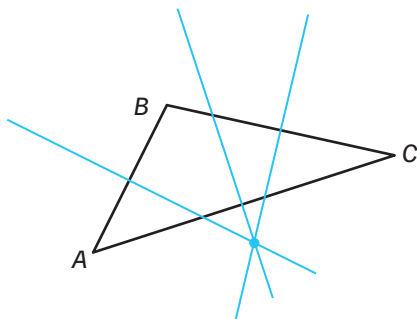
1. (a)



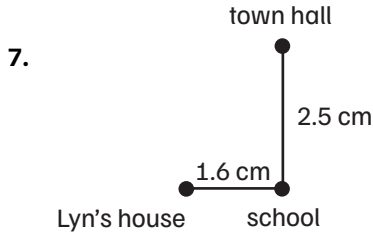
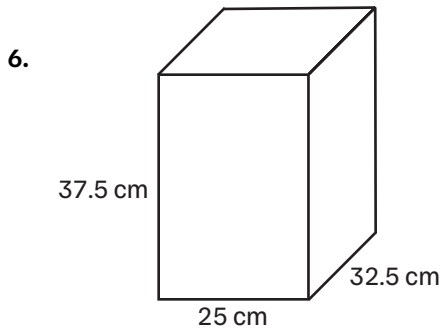
(b)



2. (a)

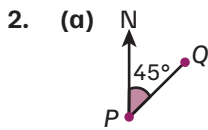


1. $66 \text{ km} = 66\,000 \text{ m} = 6\,600\,000 \text{ cm}$
 $\frac{6\,600\,000}{500\,000} = 13.2 \text{ cm}$
2. (a) $1 \text{ cm} : 20\,000 \text{ cm}$ or $1 : 20\,000$
 (b) $12.5 \text{ km} \div 2 \text{ km} = 6.25$
 $6.25 \times 10 = 62.5 \text{ cm}$
3. $12 \text{ cm} \times 50\,000 = 600\,000 \text{ cm}$
 $600\,000 \text{ cm} = 6000 \text{ m} = 6 \text{ km}$
4. $50 \text{ km} = 50\,000 \text{ m} = 5\,000\,000 \text{ cm}$
 $2 \text{ cm} : 5\,000\,000 \text{ cm} = 1 \text{ cm} : 2\,500\,000 \text{ cm}$
 $= 1 : 2\,500\,000$
5. (a) $\text{length} = \frac{60}{5} = 12 \text{ cm}$
 $\text{height} = \frac{40}{5} = 8 \text{ cm}$
 (b) $\frac{60}{7} = 8.57$
 $12 - 8.57 = 3.43 \text{ cm}$ (to 2 d.p.)



Now try these (pp 189–190)

1. (a) 040° (c) 230°
 (b) 110° (d) 330°



- (b) $180^\circ + 45^\circ = 225^\circ$
3. 012°
4. $360^\circ - 325^\circ = 35^\circ$
 $180^\circ - 35^\circ = 145^\circ$
5. (a) $360^\circ - 138^\circ = 222^\circ$
 (b) $180^\circ - 73^\circ = 107^\circ$
 $180^\circ + 107^\circ = 287^\circ$
 (c) $180^\circ - 138^\circ = 042^\circ$

Now try these (p 193)

1. (a) cuboid
 $A = 2(4 \times 3) + 2(3 \times 0.5) + 2(4 \times 0.5)$

$$= 31 \text{ cm}^2$$

- (b) triangular prism
 $A = (6 \times 12) + 2(5 \times 12) + 2\left(\frac{1}{2} \times 4 \times 6\right)$
 $= 216 \text{ cm}^2$

- (c) cylinder
 $A = (125.7 \times 61.3) + 2(\pi \times 20^2)$
 $= 10\,218.68 \text{ m}^2$

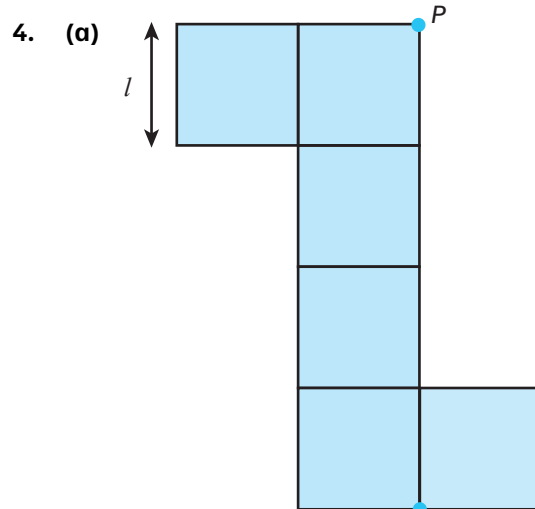
- (d) square-based pyramid
 $A = (2.3 \times 2.3) + 4\left(\frac{1}{2} \times 2.3 \times 2.95\right)$
 $= 32.43 \text{ mm}^2$

2. hexagonal prism

3. (a) cuboid

(b) $8 \times 3 \times 3 = 72 \text{ cm}^3$

(c) $A = 4(8 \times 3) + 2(3 \times 3) = 114 \text{ cm}^2$



(b) $\sqrt[3]{27} = 3 \text{ cm}$

(c) $6 \times 3 \times 3 = 54 \text{ cm}^3$

Chapter 3.4 Angle properties and facts

Now try these (pp 194–195)

1. (a) $x = 180^\circ - 40^\circ = 140^\circ$

(b) $x = 95^\circ$

(c) $x = 360^\circ - 140^\circ - 110^\circ = 110^\circ$

(d) $x = 360^\circ - 90^\circ - 55^\circ = 215^\circ$

(e) $x = 360^\circ - 30^\circ = 330^\circ$

(f) $x = 88^\circ$

(g) $x = 180^\circ - 157^\circ = 23^\circ$

(h) $x = 180^\circ - 165^\circ = 15^\circ$

2. (a) $x = 70^\circ$
opposite angles at a point are equal

- (b) $x = 110^\circ$
it is a corresponding angle to the angle adjacent to the 70° angle

OR

co-interior angles add up to 180°

3. $x = 180^\circ - 110^\circ = 70^\circ$
 $y = 180^\circ - 115^\circ = 65^\circ$
 $z = 180^\circ - 70^\circ - 65^\circ = 45^\circ$

Now try these (p 197)

1. (a) $x = 360^\circ - 78^\circ - 64^\circ - 62^\circ = 156^\circ$
(b) $x = 360^\circ - 108^\circ - 101^\circ - 51^\circ = 100^\circ$
(c) $x = (180^\circ - 63^\circ \times 2) = 54^\circ$
(d) $x = 360^\circ - 81^\circ - 12^\circ - 7^\circ = 260^\circ$
(e) $x = 136^\circ$
(f) $x = 180^\circ - (180^\circ - 90^\circ - 32^\circ) = 122^\circ$
(g) $x = \frac{360^\circ - (2 \times 128.5^\circ)}{2} = 51.5^\circ$
(h) $x = \frac{360^\circ - (2 \times 41^\circ)}{2} = 139^\circ$
2. (a) $x = \frac{360^\circ - (2 \times 75^\circ)}{2} = 105^\circ$
(b) $x = 90^\circ$

(c) $x = 360^\circ - (2 \times 94^\circ) - 15^\circ = 157^\circ$

(d) $x = \frac{360^\circ - 146^\circ - 89^\circ}{2} = 62.5^\circ$

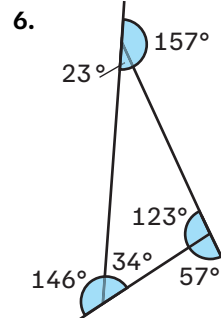
3. $\frac{360^\circ - (2 \times 99^\circ)}{2} = 81^\circ$ (both angles)

4. (a) right-angled triangle

(b) $180^\circ - 90^\circ - 60^\circ = 30^\circ$

(c) 90°

5. 70° and 110°



7. Kites have one pair of equal angles, so:
 $360^\circ - 30^\circ - 145^\circ = 185^\circ$
 $185^\circ \div 2 = 92.5^\circ$

Now try these (pp 200–201)

1. $\angle ADC = 180^\circ - 160^\circ = 20^\circ$ (opposite angles in a cyclic quadrilateral add to 180°)
2. (a) $\angle ACB = 90^\circ$ (angle subtended at circumference by a semicircle)
(b) $\angle CBA = 180^\circ - 90^\circ - 40^\circ = 50^\circ$ (angles in a triangle add to 180°)
 $\angle CBX = 90^\circ - 50^\circ = 40^\circ$ (tangent at any point is perpendicular to radius at that point)
(can also be proven by alternate segment theorem)
3. (i) $\angle OAX = 90^\circ$ (tangent at any point is perpendicular to radius at that point)

(ii) $\angle AOX = 180^\circ - 90^\circ - 30^\circ = 60^\circ$ (angles in a triangle add to 180°)

(iii) $\angle ACB = 30^\circ$ (angle subtended at the centre of a circle by an arc is twice the size of the angle subtended at the circumference by the same arc)

4. (i) $\angle AOC = 80^\circ$ (angle subtended at the centre of a circle by an arc is twice the size of the angle subtended at the circumference by the same arc)
- (ii) $\angle OAC = (180^\circ - 80^\circ) \div 2 = 50^\circ$ (equal pair of angles in isosceles triangle)
- (iii) $\angle BAC = (180^\circ - 40^\circ) \div 2 = 70^\circ$ (equal pair of angles in isosceles triangle)
 $\angle BAO = \angle BAC - \angle OAC = 70^\circ - 50^\circ = 20^\circ$
5. $x = 22^\circ$ (angles subtended by the same arc are equal)
 $y = 149^\circ$ (opposite angles are equal)
 $z = 180^\circ - 22^\circ - 149^\circ = 9^\circ$ (angles in triangle add to 180°)
6. $\angle OCA = 34^\circ$ (tangents from an external point are equal in length and $AO = BO$)
 $\angle CBO$ and $\angle CAO = 90^\circ$ (tangent at any point is perpendicular to radius at that point)
 $\angle COB$ and $\angle COA = 180^\circ - 90^\circ - 34^\circ = 56^\circ$
 $\angle DOA = 180^\circ - 56^\circ - 56^\circ = 68^\circ$ (angles on a straight line add to 180°)

Now try these (p 201)

1. $\angle DFG = \angle EDG$ (right angles)
 $\angle DGF = \angle EGD$ (shared angle)
 $\angle FDG = 180^\circ - 90^\circ - \angle DGF$
and
 $\angle DEG = 180^\circ - 90^\circ - \angle EGD$ (angles in a triangle add to 180°)
2. If parallel:
 $8x - 30 = 5y + 5$ (1) (corresponding angles are equal)
 $5y + 5 = 6x + 10$ (2) (alternate angles are equal)
(1) + (2) gives $x = 20$ and $y = 25$

$8x - 30 = 5y + 5 = 130^\circ$, so lines are parallel.
Accept other appropriate methods.

Chapter 3.5 Units and measure

Now try these (p 203)

1. (a) 3 hours and 14 minutes
(b) $(60 \times 3) + 14 = 194$ minutes
(c) 2:56 p.m.
2. (a) $(60 \times 4) + 12 = 252$ minutes
(b) $252 \times 60 = 15\,120$ seconds
3. 14:55 (2:55 p.m.)
4. (a) 32 minutes
(b) $\frac{32}{60}$ or $\frac{8}{15}$ of an hour
(c) Cecile leaves school at 16:45, which is 08:45 Canadian time.
So, they are at school at the same time for 13 minutes a day.

Now try these (p 205)

1. $86 \text{ yards} \times 0.91 = 78.26 \text{ m}$
 $100 \text{ yards} \times 0.91 = 91 \text{ m}$
 $(175 \text{ feet} \div 3) \times 0.91 = 53.08 \text{ m}$
 $(532 \text{ feet} \div 3) \times 0.91 = 161.37 \text{ m}$
Therefore, 100 yards is closest to 100 m.
2. $0.35 \times 28 = 9.8$ grams
3. (a) $12 \text{ gallons} \times 8 = 96$ pints
 $\frac{96}{1.76} = 54.55$ litres (to 2 d.p.)
(b) $\text{£}1.24 \times 54.55 \text{ litres} = \text{£}67.64$
4. (a) 5.6 km

(a) 24 km, so 15 miles

5. $1 \text{ mile} = \frac{8}{5} \text{ km}$ so 258 miles = 412.8 km
6. Owain is 74 inches tall, so about 185 cm.
Ali is 71 inches tall, so about 178 cm.
7. $200 \times \frac{1}{2.2} = 90.91 \text{ kg}$

Now try these (p 209)

1. 250 m
2. 800 kg
3. 7.5 l
4. $\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{1000}{125} = 8 \text{ m/s}$
5. (a) $\text{population density} = \frac{\text{total population}}{\text{total area}}$
 $= \frac{3187000}{20779} = 153 \text{ people per km}^2$
- (b) $\frac{20779}{14000000} = 0.00148 \text{ km}^2$
- (c) An internet search shows that a standard rugby pitch is 100 m long and 70 m wide.
This gives an area of 0.007 km^2 , which is nearly 7 times larger than the answer to (b). So, the newspaper's claim appears to be false.
6. (a) $\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{42 \text{ km}}{1.5 \text{ hours}}$
 $= 28 \text{ km/h}$
- (b) $28 \div 1.6 = 17.5 \text{ mph}$
- (c) James left at 07:20 UK time and arrived at 11:35 UK time, so the entire journey took 4 hours and 15 minutes.
Total distance = $35 + 42 + 83 = 160 \text{ km}$
 $\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{160 \text{ km}}{4.25 \text{ hours}}$
 $= 37.65 \text{ km/h}$ (to 1 d.p.)
7. $\text{rate of flow} = \frac{\text{volume}}{\text{time}} = \frac{400000 \text{ l}}{36 \text{ hours}}$
 $= \frac{400000 \text{ l}}{129600 \text{ seconds}}$
 $= 3.09 \text{ litres per second}$ (2 d.p.)

8. $\text{volume of cube} = 2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm}$
 $= 8 \text{ cm}^3$
 $\text{mass} = \text{density} \times \text{volume}$
 $\text{mass} = 22.6 \text{ g/cm}^3 \times 8 \text{ cm}^3 = 180.8 \text{ g}$

Chapter 3.6

Perimeter, area and volume

Now try these (p 210)

1. (a) $4 \times 3 \text{ cm} = 12 \text{ cm}$
- (b) $(2 \times 16 \text{ m}) + (2 \times 8 \text{ m}) = 48 \text{ m}$
- (c) $0.3 \text{ m} = 30 \text{ cm}$
 $(2 \times 50 \text{ cm}) + (2 \times 30 \text{ cm}) = 160 \text{ cm}$
- (d) $(2 \times 1 \text{ cm}) + (4 \times 0.5 \text{ cm}) = 4 \text{ cm}$

Now try these (p 213)

1. (a) $A = 5^2 = 25 \text{ cm}^2$
 $P = 4 \times 5 = 20 \text{ cm}$
- (b) $A = (7.5 \times 4) - (3 \times 1) = 27 \text{ cm}^2$
 $P = 7.5 + 4 + 4.5 + 1 + 3 + 3 = 23 \text{ cm}$
- (c) $A = \frac{1}{2} \times 22 \times 15 = 165 \text{ m}^2$
 $P = 22 + 15 + 26.6 = 63.6 \text{ m}$
- (d) $A = 12.3^2 = 151.29 \text{ m}^2$
 $P = 4 \times 12.3 = 49.2 \text{ m}$
- (e) $A = 5 \times 4 = 20 \text{ cm}^2$
 $P = (2 \times 5) + (2 \times 6) = 22 \text{ cm}$
- (f) $A = 12 \times 9 = 108 \text{ cm}^2$
 $P = (2 \times 12) + (2 \times 8) = 40 \text{ cm}$
- (g) $A = (12 \times 9) + (3 \times 2) = 114 \text{ m}^2$
 $P = 12 + 10 + 9 + 2.5 + 3 + 2.5 + 10$
 $= 49 \text{ m}$
- (h) $A = \frac{1}{2} (2 + 8) \times 7 = 35 \text{ cm}^2$
 $P = 8 + 7.6 + 2 + 7.6 = 25.2 \text{ cm}$

2. (a) trapezium

$$\begin{aligned} \text{(b) perimeter} &= 1.8 + 1.5 + 3.5 + 2.3 \\ &= 9.1 \text{ cm} \end{aligned}$$

$$\text{area} = \frac{1}{2}(1.8 + 3.5) \times 1.5 = 3.975 \text{ cm}^2$$

3. perimeter = $13 + 6 + 3 + (6 - 3) + (13 - 3) + 3$
= 38 cm

$$\text{area} = (6 \times 3) + (10 \times 3) = 48 \text{ cm}^2$$

4. area of square = $10^2 = 100 \text{ cm}^2$

$$\text{area of small triangle} = \frac{1}{2} \times 2 \times 4 = 4 \text{ cm}^2$$

$$\text{area of large triangle} = \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$$

$$\begin{aligned} \text{shaded area} &= 100 - 32 - 4 = 64 \text{ cm}^2 \\ &= 64.0\% \end{aligned}$$

5. (a) Volume of compost =

$$\begin{aligned} &\frac{1}{2} \times (380 + 165) \times 213 \times 30 \\ &= 1741(.275) \text{ (litres)} \end{aligned}$$

(b) $8 \times 0.982 \times 1.065$

Answer in the range (£)10.27 to (£)10.29

(e) $C = \pi \times 4 = 12.57 \text{ cm}$

$$A = \pi \times 2^2 = 12.57 \text{ cm}^2$$

(f) $C = \left(\frac{1}{2} \times \pi \times 5\right) + 5 = 12.85 \text{ cm}$

$$A = \frac{1}{2}(\pi \times 2.5^2) = 9.82 \text{ cm}^2$$

(g) $C = \left(\frac{1}{2} \times \pi \times 7.1\right) + 7.1$

$$= 18.25 \text{ mm}$$

$$A = \frac{1}{2}(\pi \times 3.55^2) = 19.80 \text{ cm}^2$$

(h) P (top) = $\left(\frac{1}{2} \times \pi \times 108\right) + (108 - 62)$
= $(54\pi + 46) \text{ m}$

$$\begin{aligned} P \text{ (bottom)} &= \left(\frac{1}{2} \times \pi \times 62\right) \\ &= 31\pi \text{ m} \end{aligned}$$

$$\begin{aligned} \text{total perimeter} &= (54\pi + 46) + 31\pi \\ &= 313.04 \text{ m} \end{aligned}$$

$$\begin{aligned} A \text{ (top)} &= \frac{1}{2}(\pi \times 54^2) \\ &= 1458\pi \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A \text{ (bottom)} &= \frac{1}{2}(\pi \times 31^2) \\ &= 480.5\pi \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{total area} &= 1458\pi + 480.5 \\ &= 5060.94 \text{ m}^2 \end{aligned}$$

Now try these (p 214)

All answers given to 2 decimal places.

1. (a) $C = \pi \times 6.5 = 20.42 \text{ mm}$

$$A = \pi \times 3.25^2 = 33.18 \text{ mm}^2$$

(b) $P = \left(\frac{1}{2} \times \pi \times 3\right) + (3 \times 3)$

$$= 13.71$$

$$A = \left(\frac{1}{2} \times \pi \times (1.5)^2\right) + 3^2$$

$$= 12.53 \text{ m}^2$$

(c) $C = \pi \times 10 = 31.42 \text{ cm}$

$$A = \pi \times 5^2 = 78.54 \text{ cm}^2$$

(d) $P = \left(\frac{1}{2} \times \pi \times 1.8\right) + (2 \times 0.35) + 1.8$

$$= 5.33 \text{ m}$$

$$A = \left(\frac{1}{2} \times \pi \times 0.9^2\right) + (1.8 \times 0.35)$$

$$= 1.90 \text{ m}^2$$

2. P (left) = $\left(\frac{1}{2} \times \pi \times 35\right) = 17.5\pi \text{ cm}$

$$P \text{ (middle)} = (2 \times 34) + (35 - 20) = 83 \text{ cm}$$

$$P \text{ (right)} = \left(\frac{1}{2} \times \pi \times 20\right) = 10\pi \text{ cm}$$

$$\begin{aligned} \text{total perimeter} &= 17.5\pi + 83 + 10\pi \\ &= 169.39 \text{ cm} \end{aligned}$$

$$A \text{ (left)} = \left(\frac{1}{2} \times \pi \times 17.5^2\right) = 153.125\pi \text{ cm}^2$$

$$A \text{ (middle)} = (34 \times 35) = 1190 \text{ cm}^2$$

$$A \text{ (right)} = \left(\frac{1}{2} \times \pi \times 10^2\right) = 50\pi \text{ cm}^2$$

$$\begin{aligned} \text{total area} &= 153.125\pi + 1190 + 50\pi \\ &= 1828.14 \text{ cm}^2 \end{aligned}$$

Now try these (p 215)

All answers given to 2 decimal places.

1. (a) $\frac{1}{2}\pi$ cm
 (b) $\frac{3}{4}\pi$ cm²
 (c) $\frac{3}{4}\pi - \frac{9}{4}$ cm²
 (d) arc length = 1.6 cm
 sector area = 22.4 cm²
 segment area = 0.1 cm²
2. 4π cm², 12.6 cm²
3. (a) $\frac{\theta}{360} \times \pi \times 8 = 5$ cm
 $\frac{\theta}{360} = \frac{5}{8\pi}$
 $\theta = 360 \left(\frac{5}{8\pi} \right)$
 $\theta = 71.6^\circ$ (to 1 d.p.)
 (b) 6.98 cm² (to 1 d.p.)

Now try these (p 219)

1. (a) $A = \left(\frac{1}{2} \times 4 \times 3 \right) + (6 \times 4) = 30$ cm²
 (b) $V = \text{cross-sectional area} \times \text{length}$
 $= 30 \times 7 = 210$ cm³
2. $A = 2(40 \text{ cm} \times 55 \text{ cm}) + 4(80 \text{ cm} \times 55 \text{ cm})$
 $= 22\,000$ cm² or 2.2 m²
 $V = 55 \text{ cm} \times 40 \text{ cm} \times 80 \text{ cm}$
 $= 176\,000$ cm³ or 0.176 m³
3. V (large) = $(\pi \times 5^2 \times 30) = 750\pi$ cm³
 V (small) = $(\pi \times 3^2 \times 30) = 270\pi$ cm³
 total volume = $750\pi - 270\pi$
 $= 1507.96$ cm²
4. A (hemisphere) = $\frac{1}{2} \times 4 \times \pi \times r^2$
 $= 18\pi$ cm²
 A (cone) = $\pi \times r \times l$
 $= (9\sqrt{7})\pi$ cm²
 A (shape) = $(18 + 9\sqrt{7})\pi$ or
 $= 173.13$ cm² (to 2 d.p.)
 V (hemisphere) = $\frac{1}{2} \times \frac{4}{3} \times \pi \times r^3$
 $= 18\pi$ cm³

$$V(\text{cone}) = \frac{1}{3} \times \pi \times r^2 \times h$$

$$= 36\pi \text{ cm}^3$$

$$V(\text{shape}) = 54\pi \text{ cm}^3$$

$$= 169.65 \text{ cm}^3 \text{ (to 2 d.p.)}$$

5. (a) $V = \pi \times r^2 \times h$
 $= 0.31$ m³ (to 2 d.p.)
- (b) total $A = A$ of larger cylinder – opening
 A (large) = $2(\pi \times r^2) + (\pi \times 2r \times h)$
 $= 3.9309\dots$ m²
 A (opening) = $\pi \times r^2 = 0.1590\dots$ m²
 A (tank) = $3.7718\dots$ m² = 3.77 (to 2 d.p.)

Chapter 3.7 Pythagoras' theorem

Now try these (p 221)

1. (a) $a^2 + b^2 = c^2$
 $12^2 + 9^2 = c^2$
 $225 = c^2$
 $\sqrt{225} = c$
 $15 = c$
 So, missing side = 15 cm
- (b) $a^2 + b^2 = c^2$
 $24^2 + b^2 = 26^2$
 $576 + b^2 = 676$
 $b^2 = 676 - 576$
 $b^2 = 100$
 $b = 10$
 So, missing side = 10 cm
- (c) $a^2 + b^2 = c^2$
 $7^2 + b^2 = 25^2$
 $49 + b^2 = 625$
 $b^2 = 625 - 49$
 $b^2 = 576$
 $b = 24$
 So, missing side = 24 cm

2. $a^2 + b^2 = c^2$
 $4^2 + b^2 = 10^2$
 $16 + b^2 = 100$
 $b^2 = 100 - 16$
 $b^2 = 84$
 $b = 9.17$
 So, perpendicular height = 9.17 cm (to 2 d.p.)
3. $10^2 + 6^2 = 136$
 $\sqrt{136} = 11.66$ cm
4. (a) $8^2 + 6^2 = 100$
 $\sqrt{100} = 10$ cm
 yes, this triangle is right-angled
- (b) $2^2 + 1.5^2 = 6.25$
 $\sqrt{6.25} = 2.5$ cm
 yes, this triangle is right-angled
- (c) $4^2 + 6^2 = 36$
 $\sqrt{52} = 7.21$ cm
 no, this triangle is not right-angled

Now try these (p 224–225)

All answers given to 2 decimal places.

1. (a) 22.66 cm (d) 3.26 m
 (b) 1.55 cm (e) 4.39 m
 (c) 25.39 m
2. (a) 17.92° (d) 27.90°
 (b) 30.26° (e) 54.92°
 (c) 53.27°
3. $PR = 5.7125...$ cm
 $PQ = 13.52$ cm (to 2 d.p.)

Now try these (p 226)

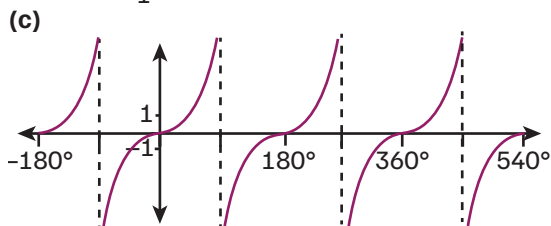
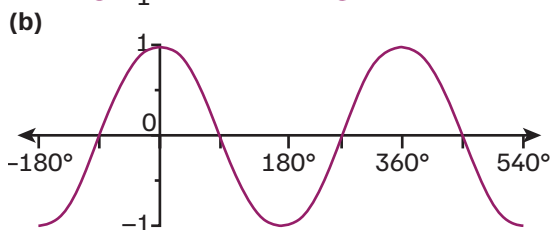
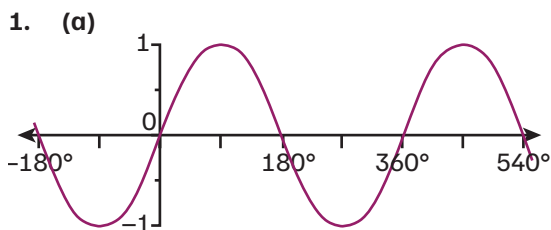
1. $\sin(15) = \frac{180}{x}$
 $x = 695.47$ m (to 2 d.p.)

Now try these (p 228)

All answers given to 2 decimal places.

1. (a) 23.96°
 (b) 43.29 cm
 (c) 6.77 mm
2. (a) $\text{Area} = \frac{1}{2} ab \sin C$
 $\text{Area} = \frac{1}{2} \times 185 \text{ m} \times 215 \text{ m} \times \sin 65^\circ$
 $\text{Area} = 18\,024.20 \text{ m}^2$
- (b) $AC^2 = 215^2 + 185^2 - 2(215)(185)(\cos 65^\circ)$
 $AC^2 = 46\,830.7172...$ m
 $AC = 216.4040...$ m
 find $\angle ADC$:
 $(216.4040...)^2 = 134^2 + 190^2 -$
 $2(134)(190)(\cos \theta)$
 $\theta = 81.8424...$ °
 $\text{Area} = 18\,024.20 \text{ m}^2 + \left(\frac{1}{2} \times 134 \times 190 \times \sin 81.8424...^\circ\right)$
 $= 30\,625.3946...$ m²
 $= 30\,625.40 \text{ m}^2$ (to 2 d.p.)
3. DF is the hypotenuse of triangle DAF
 $AF = 1.9209...$ m (Pythagoras' theorem)
 $DA = 0.8$ m (given)
 $DF = \sqrt{1.9209^2 + 0.8^2} = 2.0808...$ m
 $\angle DFA = \sin^{-1}\left(\frac{0.8}{2.0808...}\right) = 22.61^\circ$
4. (a) $CX = \sqrt{3^2 + 3^2} = 3\sqrt{2}$
 $CV = \sqrt{(3\sqrt{2})^2 + 10^2} = \sqrt{118}$ cm
 $= 10.86$ cm (to 2 d.p.)
- (b) $\angle ACV = \angle XCV$
 $\angle XCV = \sin^{-1}\left(\frac{10}{\sqrt{118}}\right) = 67.01^\circ$

Now try these (p 229)



2. $233.1^\circ, 306.9^\circ, 593.1^\circ, 666.9^\circ$
3. $-71.0^\circ, 109.0^\circ, 289.0^\circ, 469.0^\circ$
4. (a) $75.5^\circ, 284.5^\circ, 435.5^\circ, 644.5^\circ$
 (b) $138.6^\circ, 221.4^\circ, 498.6^\circ, 581.4^\circ$
 (c) $197.5^\circ, 342.5^\circ, 557.5^\circ, 702.5^\circ$
 (d) $6.8^\circ, 186.8^\circ, 366.8^\circ, 546.8^\circ$
 (e) $231.3^\circ, 208.7^\circ, 591.3^\circ, 668.7^\circ$
 (f) $116.6^\circ, 296.6^\circ, 476.6^\circ, 656.6^\circ$

Chapter 3.8 Position, symmetry and transformations

Now try these (p 231)

1. (a) $\left(\frac{4+6}{2}, \frac{2+8}{2}\right) = (5, 5)$
 (b) $\left(\frac{1+3}{2}, \frac{2+4}{2}\right) = (2, 3)$

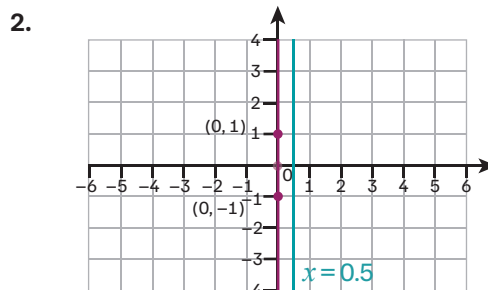
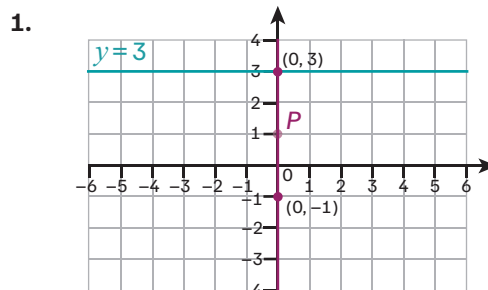
(c) $\left(\frac{7+(-3)}{2}, \frac{1+0}{2}\right) = (2, 0.5)$

(d) $\left(\frac{-2.5+(-7)}{2}, \frac{5-10}{2}\right) = (-4.75, 7.5)$

(e) $\left(\frac{10+(-20)}{2}, \frac{1+5}{2}\right) = (-5, 3)$

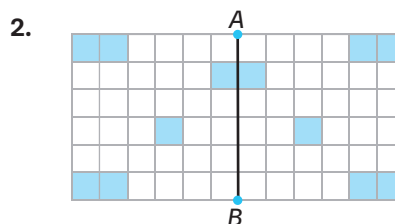
2. $(0, -2)$

Now try these (p 233)



Now try these (p 235)

1. (a) E, K, T, W
 (b) H, X



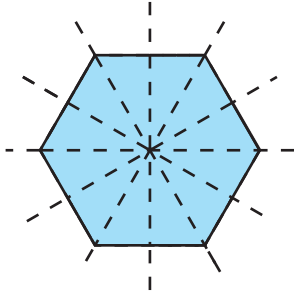
3. (a) hexagon
 Note: this is not a regular hexagon as

the horizontal sides are 2 squares long but the diagonal sides are longer. This is because the diagonal of any square is longer than its side lengths.

(b) 2 lines of symmetry

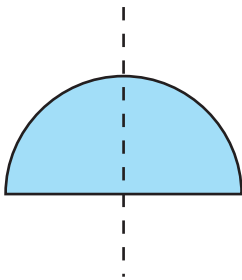
(c) 2

4. (a) Order of rotational symmetry = 6

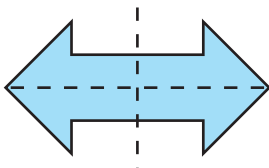


(b) No lines of symmetry. Order of rotational symmetry = 2

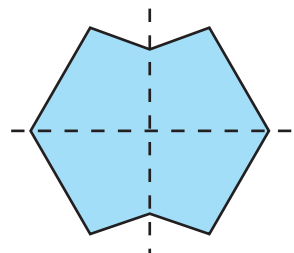
(c) Order of rotational symmetry = 1



(d) Order of rotational symmetry = 2

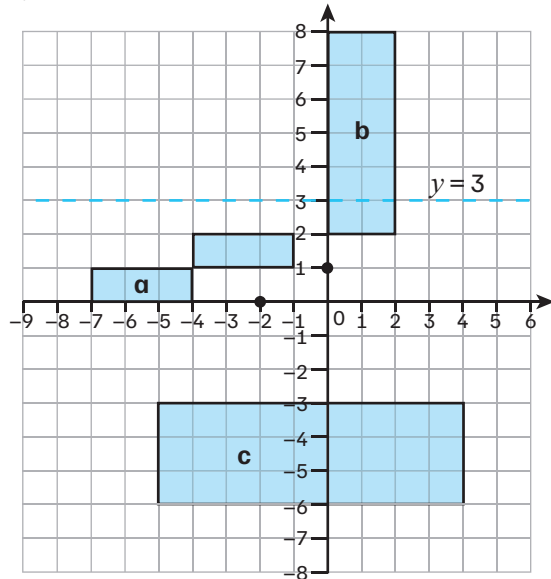


(e) Order of rotational symmetry = 2



Now try these (p 241)

1.



2. rotation of 180° , centre (0, 2)

enlargement by scale factor -1 , centre (0, 2)

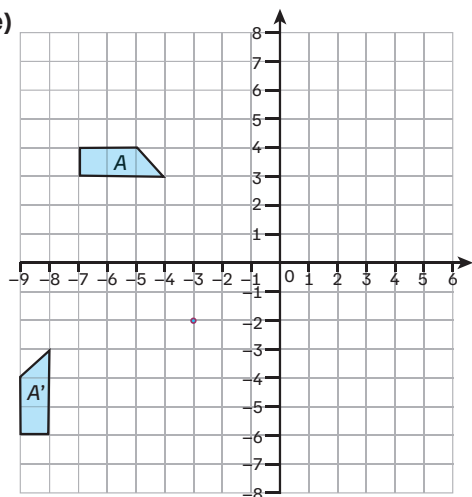
3. (a) translation of 10 squares right and 1 square down

(b) rotation of 180° , centre (3, 1)

(c) enlargement of scale factor 2, centre (3, 3)

(d) translation of 2 squares right and 5 squares down, reflection in the line $x = -3$ (Accept other suitable answers.)

(e)



- (f) Enlargement by scale factor -2 , centre $(-3\frac{2}{3}, 1)$.

Chapter 3.9 Congruency and similarity

Now try these (p 243)

1. A and F
B and J
C and H
E and G
2. smaller triangle: $24 - 8 - 6 = 10$ cm
larger triangle: $24 - 10 - 6 = 8$ cm
yes, they are congruent
3. Yes, the quadrilaterals can be congruent, since the missing angles in A must add to 180° , so could both be 90° , while the missing angles in B must also add to 180° , so could be 95° and 85° .
4. (a) 0.4
(b) $x = 2.5$ cm
5. Yes. Each side of the larger triangle is 4 times larger than the corresponding side in the smaller triangle.
6. Yes. Interior angles of a triangle add up to 180° , meaning the missing angle of the smaller triangle is 15° and the missing angle of the larger triangle is 75° . Therefore, both triangles have the same interior angles and are similar.
7. Scale factor = 2
 $y = 8$ cm
8. (a) Each side of the larger quadrilateral is 1.5 times larger than the corresponding side in the smaller quadrilateral.
(b) $6 \div 1.5 = 4$ cm

9. (a) The triangles are similar because each side of the larger triangle is exactly 1.5 times bigger than the corresponding side of the smaller triangle.
(b) $w = 180^\circ - 75^\circ - 30^\circ = 75^\circ$
(c) The triangles are isosceles, so angle w must be the same as the 75° angle.

Now try these (p 244)

1. Scale factor = $12 \div 5 = 2.4$
 $x = 8 \times 2.4 = 19.2$ cm
2. 392 cm²
3. 7 cm
4. 2000 cm³

Now try these (p 245)

1. (a) Yes. Both triangles have a right angle and a hypotenuse of 20 cm, as well as a side of 10 cm, so they are congruent (RHS).
(b) No. The known angle is not between the known sides, so we cannot be sure that these triangles are congruent.
(c) Yes. Both triangles have a 55° angle between a 5 cm side and a 3 cm side, so they are congruent (SAS).
(d) No. The triangles share the same angles but their sides could be different lengths, which would make them similar rather than congruent.

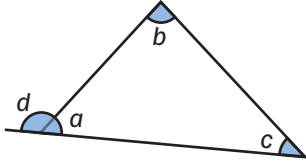
Revision round up (p 246)

1. 800 cm \div 4 cm = 200, so 1 : 200
2. 270°
3. $a = 42^\circ$
 $b = 65^\circ$
 $c = 180^\circ - 65^\circ = 115^\circ$

4. $\angle PQR = 180^\circ - 101^\circ = 79^\circ$ (opposite angles in a cyclic quadrilateral add to 180°).
5. $\angle ABC = 58^\circ$ (alternate segment theorem).
 $\angle BAC = 90^\circ$ (angle subtended at circumference by semicircle is always a right angle).
 $\angle ACB = 180^\circ - 58^\circ - 90^\circ = 32^\circ$ (angles in a triangle add to 180°).

6. (a) scale factor -0.5
 centre of enlargement $(3, 2)$
 (b) Rotate 90° anticlockwise using a centre of rotation of $(7, 3)$. Then, translate 4 units left. (Accept other suitable answers.)

7.



$$d + a = 180^\circ \text{ (angles on a straight line)}$$

$$\text{so, } d = 180^\circ - a$$

$$a + b + c = 180^\circ \text{ (angles in a triangle)}$$

$$\text{so, } b + c = 180^\circ - a$$

$$\text{so, } b + c = d$$

$$\begin{aligned} 8. \quad V(\text{large}) &= \frac{1}{3}\pi r^2(4h) \\ &= \frac{1}{3}\pi(12)^2(4h) \\ &= 192\pi h \end{aligned}$$

$$\begin{aligned} V(\text{small}) &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(3)^2 h \\ &= 3\pi h \end{aligned}$$

$$\begin{aligned} V(\text{frustum}) &= 192\pi h - 3\pi h \\ &= 189\pi h \end{aligned}$$

9. (a) speed = $2.4 \text{ km} \div 0.5 \text{ hours} = 4.8 \text{ km/h}$
 distance = $4.8 \text{ km/h} \times 0.75 \text{ hours}$
 $= 3.6 \text{ km}$

(b) length of route = $2.4 + 3.6 = 6 \text{ km}$

$$\text{speed} = 6 \text{ km} \div \frac{2}{3} = 9 \text{ km/h}$$

10. (a) $(3, -1)$

(b) $(-5, 1)$

(c) $(-0.5, -0.5)$

11. $BC = 8 \text{ cm}$

$$AB = \sqrt{8^2 + 6^2} = 10 \text{ cm}$$

12. (a) $x = 60^\circ$

(b) 3 sides

13. (a) 1530 ml

(b) $1.53 \text{ l} \div 4.5 = 0.34 \text{ gallons.}$

(c) 1 cl = 10 ml

$$1530 \text{ ml} = 153 \text{ cl}$$

$$153 \div 17.5 = 8.74\dots = 8 \text{ full cups}$$

14. (a) $74.6^\circ, 105.4^\circ, 434.6^\circ, 465.4^\circ$ (to 1 d.p.)

(b) $72.9^\circ, 252.9^\circ, 432.8^\circ, 612.8^\circ$ (to 1 d.p.)

15. $\angle ABD = 90^\circ$ (tangent at any point is perpendicular to radius at that point)

So, triangle is right-angled.

$$AB = 12 \div \tan 55^\circ = 8.4024\dots \text{ cm}$$

$$\begin{aligned} \text{Area of sector} &= \frac{55}{360} \times \pi \times (8.4024\dots)^2 \\ &= 33.8864\dots \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of triangle} &= 0.5 \times 8.4024\dots \times 12 \\ &= 50.4149\dots \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Shaded area} &= 50.414\dots \text{ cm}^2 - 33.886\dots \text{ cm}^2 \\ &= 16.5 \text{ cm}^2 \text{ (to 1 d.p.)} \end{aligned}$$

16. (a) Volume scale factor = $125 \div 8 = 15.625$

$$\text{Linear scale factor} = \sqrt[3]{15.625} = 2.5$$

So, radius of A : radius of B = 1 : 2.5

(b) Area scale factor = $2.5^2 = 6.25$

$$40 \times 6.25 = 250 \text{ cm}^2$$

17. (a) $x + 9x = 180^\circ$

$x = 18^\circ$

interior angle $= 9 \times 18^\circ = 162^\circ$

(b) exterior angle $= 18^\circ$

$360^\circ \div 18^\circ = 20$ sides

(c) 360°

18. $\angle DFG = \angle EDG$ (right angles)

$\angle DGF = \angle EGD$ (shared angle)

$\angle FDG = 180^\circ - 90^\circ - \angle DGF$

and

$\angle DEG = 180^\circ - 90^\circ - \angle EGD$ (angles in a triangle add to 180°)

So yes, the triangles are similar because they share the same angles.

19. $\frac{1}{2} \times 12 \times AD \times \sin 72^\circ = 56$

$AD = 9.8136\dots$ cm

$AC^2 = 12^2 + (9.81\dots)^2 - 2(12)(9.81\dots)(\cos 72^\circ)$

$AC = 12.9431\dots$ cm

$\frac{12.9431\dots}{\sin 82} = \frac{BC}{\sin 59}$

$BC = 11.2034\dots$ cm

$\angle BCA = 180^\circ - 59^\circ - 82^\circ = 39^\circ$

Area of $ABC = \frac{1}{2} \times 12.9431\dots \times 11.2034\dots$

$\times \sin 39^\circ$

$= 45.63$ cm² (to 2 d.p.)

20. (a) If SPM is a triangle, $\angle SPM = 360^\circ - 145^\circ - 110^\circ = 105^\circ$

$p^2 = s^2 + m^2 - 2(s)(m)(\cos P)$

$p = 16.05$ km (to 2 d.p.)

(b) Using the Sine rule:

$\frac{16.05}{\sin 105^\circ} = \frac{8}{\sin M}$

$M = 28.7751\dots^\circ$

So, bearing of S from M =

$360^\circ - 28.7751^\circ = 331.22^\circ$ (to 2 d.p.)

21. (a) Exposed SA of cylinder:

$\pi r^2 + 2\pi r h = 48\pi$ cm²

Slant height of cone $= \sqrt{3^2 + 6^2}$
 $= 3\sqrt{5}$ cm

Exposed SA of cone:

$\pi r l = (9\sqrt{5})\pi$ cm²

SA of shape $= (48 + 9\sqrt{5})\pi$ cm²

V of cylinder:

$\pi r^2 h = 58.5\pi$ cm³

V of cone:

$\frac{1}{3} \pi r^2 h = 18\pi$ cm³

V of shape $= 76.5\pi$ cm³

(b) $\pi(3)^2 h + 18\pi = 200$

$h = \frac{200 - 18\pi}{9\pi}$

$h = 5.0735\dots = 5.07$ cm (to 2 d.p.)

22. $SU = 9$ cm (same as PQ)

$\angle TUS = \angle TSU = 82^\circ$

$UT = 4.5$ cm $\div \cos 82^\circ$

$= 32.3338\dots$ cm

$\angle UPT = \angle TQS = 29^\circ$

$PT = 32.3338\dots$ cm $\div \sin 29^\circ$

$= 66.6938\dots$ cm $= 69.69$ cm (to 2 d.p.)

23. linear scale factor $= \frac{23}{12}$

volume scale factor $= \left(\frac{23}{12}\right)^3 = \frac{12167}{1728}$
 $= 7.0410\dots$

volume of small bottle $= 4 \div 7.0410\dots$

$= 0.5680\dots$ pints

$0.5680\dots$ pints $\times 0.57 = 0.3238\dots$ l

$= 323.8$ ml (to 1 d.p.)

Section 4: Statistics and Probability

Chapter 4.1 Data handling cycle – collecting data

Now try these (p 255)

- (a)** Accept any suitable answer such as:

Random sampling (described method of using a random number generator, random number table, or lottery system).

Systematic sampling (described method of selecting every 3rd person).

Stratified sampling (described method of splitting employees into strata such as sex or department, and then selecting participants within each strata).

(b) Stratified sampling

(c) 1. Split employees into two strata: desk-based and deskless.

2. Work out number of people to select from each strata using the formula:

$$\frac{\text{strata size}}{\text{total population}} \times 6$$

3. Use a suitable sampling method to select participants within each strata.

4. If the final sample size is too small or too large, decide which strata should have its number of participants rounded up/down.
- Bio: 1st year = 7, 2nd year = 7,
3rd year = 6

Chem: 1st year = 11, 2nd year = 10,
3rd year = 10

This leads to sample size of 51, so round down Biology 2nd year to 6 students.
- (a)** 1400

(b) Stratified sampling
- (a)** Jonah should consider whether he can investigate everyone at the school (which would involve recording how long it takes each student to finish their homework and how many classes they have missed).

He would also need to consider whether he assesses the completion time of just one homework for one subject, or multiple.

A smaller sample size might be useful, such as a random group of students from each year group.

(Accept any other suitable answer)

(b) Jonah should consider sample size, the sample representation (i.e. whether all years and abilities are represented), and the method of data collection (i.e. will students complete the homework at home and measure the time taken or will they complete it in class?).

(Accept any other suitable answer)
- (a)** 20 and under: 67
20 to 60: 331
over 60: 101

This leads to a sample size of 499, so round up the 'over 60' group to 102 participants.

(b) Accept any suitable answers. Examples include:

The hypothesis focuses ‘young’ people and ‘older’ people, but the sample includes all age groups.

The age groups are very broad, so age ranges typically considered ‘young’ and ‘older’ are split across multiple groups (e.g. 25 years old is generally considered young, but would be counted within the middle age group).

Older people are more likely to use healthcare services than younger people, so many young people selected in the sample will not be representative of young people who use healthcare services.

(c) 1. Find a list of people who already use healthcare services to select the sample from.

2. Group members of the population into age groups (e.g. by decade) and define which are ‘young’ and ‘older’. Remove groups that are not considered ‘young’ or ‘older’.

3. Use stratified sampling to select participants from each age group to gain a better insight into how opinions towards healthcare services vary with age.

Now try these (p 257)

- (a)** Students in Year 7 and 8 have likely taken fewer tests than older students.
- (b)** If they have already bought an item, their in-store experience is likely to have been positive.

(Accept any other suitable answer)

(c) People using social media are probably already regular users of the internet.

(Accept any other suitable answer)

2. **(a)** “How does our milk brand compare to others on the market?”

(Accept any other suitable answer)

(b) “How often do you clean your fish tank?”

(Accept any other suitable answer)

(c) “How would you rate your experience with our staff?”

(Accept any other suitable answer)

(d) “How could our products be improved?”

(Accept any other suitable answer)

Now try these (p 260)

1.

| Mark $p\%$ | Tally | Frequency |
|-------------------|-------|-----------|
| $30 \leq p < 40$ | | 1 |
| $40 \leq p < 50$ | | 3 |
| $50 \leq p < 60$ | | 3 |
| $60 \leq p < 70$ | | 1 |
| $70 \leq p < 80$ | | 3 |
| $80 \leq p < 90$ | | 3 |
| $90 \leq p < 100$ | | 1 |

2.

| Colour | Tally | Frequency |
|--------|-------|-----------|
| Red | | 2 |
| Grey | | 6 |
| Black | | 3 |
| White | | 2 |
| Blue | | 1 |
| Yellow | | 1 |

Now try these (p 263)

1. **(a)** The missing number is:
 $50 - 4 - 12 - 20 - 5 = 9$

(b) Quantitative continuous data

(c)

| t minutes | Frequency |
|------------------|-----------|
| $10 \leq t < 20$ | 4 |
| $20 \leq t < 30$ | 12 |
| $30 \leq t < 35$ | 15 |
| $35 \leq t < 40$ | 5 |
| $40 \leq t < 50$ | 9 |
| $50 \leq t < 60$ | 5 |

2. (a)

| height h cm | Frequency |
|--------------------|-----------|
| $100 \leq h < 150$ | 3 |
| $150 \leq h < 200$ | 8 |
| $200 \leq h < 250$ | 9 |
| $250 \leq h < 300$ | 3 |
| $300 \leq h < 350$ | 3 |
| $350 \leq h < 400$ | 4 |

Note: the tree with a height of 400 cm does not fall into any of these categories.

(b) For example:

Advantage: grouping data makes it easier to spot patterns, such as the most and least common height.

Disadvantage: Doesn't give exact values so doesn't give an idea of where each height lies within each class interval.

(Accept any other suitable answer)

3. (a) Qualitative

(b) Quantitative continuous

(c) Qualitative

(d) Qualitative

(e) Quantitative discrete

(f) Quantitative continuous

(g) Quantitative discrete

(h) Quantitative continuous

(i) Qualitative

4. (a) For example, the colour of someone's shoes.

(b) For example, number of pencils in a pencil case.

(c) For example, the height of classmates.

5. (a)

| Age y years | Frequency |
|------------------|-----------|
| $0 \leq y < 10$ | 8 |
| $10 \leq y < 20$ | 10 |
| $20 \leq y < 30$ | 7 |
| $30 \leq y < 40$ | 2 |
| $40 \leq y < 50$ | 2 |
| $50 \leq y < 60$ | 1 |

(b)

| Age y years | Frequency |
|------------------|-----------|
| $0 \leq y < 5$ | 3 |
| $5 \leq y < 10$ | 5 |
| $10 \leq y < 15$ | 5 |
| $15 \leq y < 20$ | 5 |
| $20 \leq y < 40$ | 9 |
| $40 \leq y < 60$ | 3 |

Accept different class intervals.

Chapter 4.2 Data handling cycle - results

Now try these (pp 265-266)

1. Ordered list: 1, 1, 1, 3, 3, 3, 4, 5, 6

$$\text{Mean} = (1 + 1 + 3 + 5 + 1 + 3 + 3 + 6 + 4) \div 9 = 3$$

$$\text{Median} = 3$$

$$\text{Mode} = 1 \text{ and } 3$$

2. Ordered list = 1, 1, 1, 1, 2, 4, 5, 7, 10, 10, 10, 10, 10, 10, 10

(a) Mean = 6.13 (to 2 d.p.)

$$\text{Median} = 7$$

$$\text{Mode} = 10$$

(b) The mode would be best for advertising the restaurant because it is the highest.

3. Ordered list = 0, 0, 0, 0, 0, 1, 2, 2, 3, 20

(a) Mean: 2.8

$$\text{Median: } 0.5$$

$$\text{Mode: } 0$$

(b) The median best represents the data because most of the households have 3 or fewer pets. The mean has been skewed by the unusually high number of pets in the last house.

4. Ordered list = 1, 2, 2, 2, 2, 4, 4, 4, 5, 5, 7, 8

(a) 46

(b) Mean = 3.83 (to 2 d.p.)

Median = 4

Mode = 2

5. Group A total score = $25 \times 45 = 1125$

Group B total score = $30 \times 50 = 1500$

Total score of both groups = $1125 + 1500$
 $= 2625$

Total number of students in both groups
 $= 25 + 30 = 55$

Mean of combined groups
 $= 2625 \div 55 = 47.7272\dots = 47.73$ (to 2 d.p.)

Now try these (p 268)

1. (a)

| Number of people | Tally | Frequency | $f \times x$ |
|------------------|-------|-----------|--------------|
| 0 | | 3 | 0 |
| 1 | | 2 | 2 |
| 2 | | 8 | 16 |
| 3 | | 2 | 6 |
| 4 | | 3 | 12 |
| 5 | | 2 | 10 |
| 6 | | 4 | 24 |

(b) Mean = $(0 + 2 + 16 + 6 + 12 + 10 + 24) \div$
 $(3 + 2 + 8 + 2 + 3 + 2 + 4)$
 $= 70 \div 24$

$= 2.92$ (2 d.p.)

Median = 2

Mode = 2

Now try these (pp 271)

1. There are many possible answers.

For example: 3, 5, 5, 5, 7

To have a mean of 5, the numbers must add to 25

To have a mode of 5, the numbers must have more 5s than any other number

To have a median of 5, the middle value

must be 5 when the numbers have been ordered.

2. **A:** False. The median is the centre value and there is only one centre value.

B: False. If there is an even number of values the median is the midpoint of the middle two values.

C: True.

D: True.

E: False. As the mean is calculated by adding the values and dividing by how many there are, it does not matter what order the numbers are in.

3. sum of all values = $4 \times 9 = 36$

missing number = $36 - 3 - 7 - 10 = 16$

4. (a) Ordered list: 2, 2, 4, 5, 8, 10, 13

Mean = $44 \div 7 = 6.2857\dots$
 $= 6.3$ to 1 d.p.

Median = 5

Mode = 2

(b) These statistics show that there is a lot of variation in the number of texts Luca receives. The mean is higher than the median, suggesting Luca received a high number of texts on some days. The mode is lower than the median, suggesting Luca usually doesn't receive many texts.

5. (a) $90 \leq w < 100$

(b) total $f \times w = 1950$

total $f = 22$

estimated mean = $1950 \div 22$

$= 88.6363\dots$

$= 88.64$ kg (to 2 d.p.)

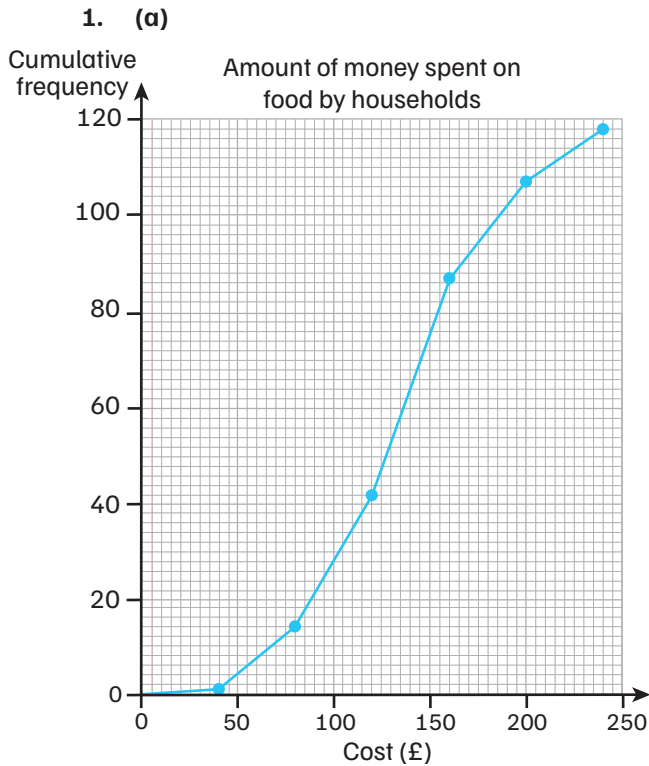
(c) median is in class $90 \leq w < 100$

position of median = $\frac{11-10}{8}$

$= 12.5\%$ along class

median = $90 + (0.125 \times 10) = 91.25$ kg

Now try these (pp 274)



(b) £135

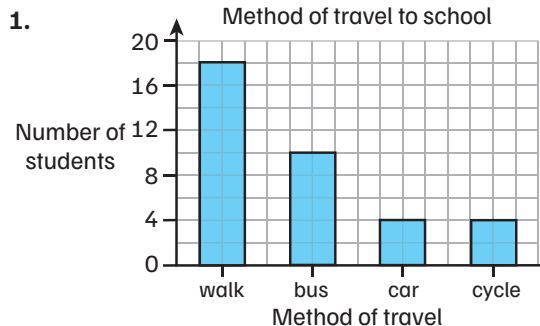
(c) $Q1 = £103$, $Q3 = £163$

IQR = £60

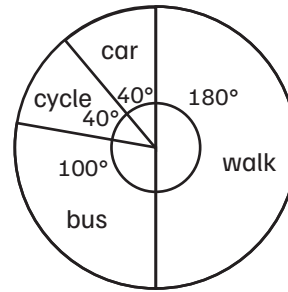
(d) Cumulative frequency at £50 = 4

So, % who spend less than £50 = 3.4%
(to 1 d.p.)

Now try these (pp 278)

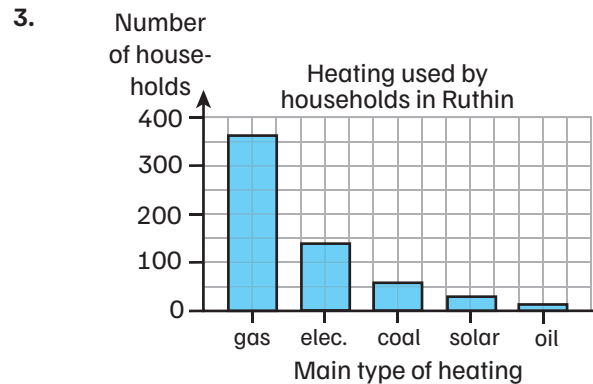


Methods of travel to school



2. (a) Tegid could be correct because there could be the same number of students attending chess and badminton clubs. This would mean that the portion of Year 10 students who play chess represents more people than the portion of Year 10 students who play badminton.

(b) Tegid could be incorrect because there could be more students that play badminton than chess, meaning the Year 10 portion who play chess could represent fewer students than the smaller Year 10 portion who play badminton.



4. (a) $144 \div 24 = 6$ sandwiches
(b) $6 \times 3 = 18$ sandwiches

Now try these (p 281)

1. (a)

| Mass (m kg) | Midpoint | Frequency |
|------------------|----------|-----------|
| $0 \leq m < 10$ | 5 | 7 |
| $10 \leq m < 20$ | 15 | 15 |
| $20 \leq m < 30$ | 25 | 41 |
| $30 \leq m < 40$ | 35 | 24 |
| $40 \leq m < 50$ | 45 | 2 |

(b) total = 89, median = 44.5th dog

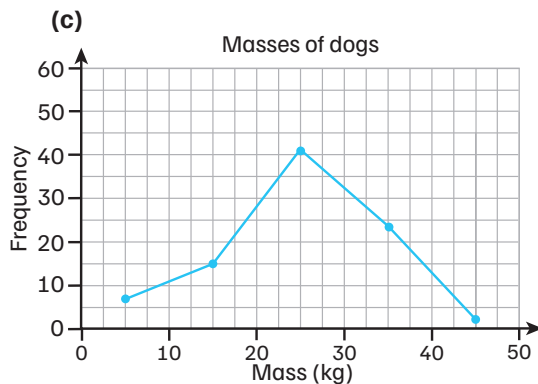
median is in class $20 \leq m < 30$

$$\text{position of median} = \frac{44.5 - 22}{41}$$

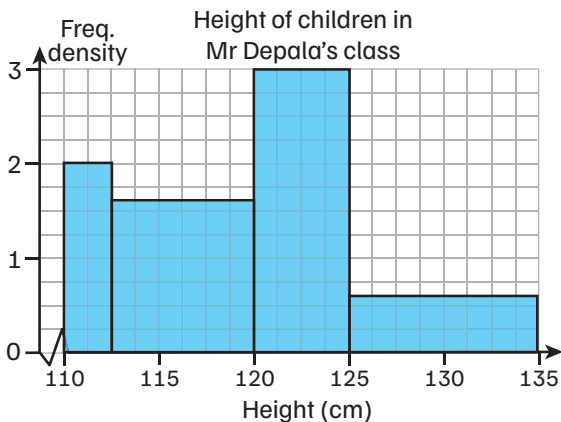
$$= 54.8\% \text{ along class}$$

$$\text{median} = 20 + (0.548 \times 10)$$

$$= 25.5 \text{ kg (to 1 d.p.)}$$



2. (a)



(b) $Q1 = 9.5\text{th child}$

$$w \times 1.6 = 4.5, w = 2.8125$$

$$Q1 = 115.3125 \text{ cm}$$

$Q3 = 28.5\text{th child}$

$$w \times 3 = 11.5, w = 3.83\ldots$$

$$Q3 = 123.8 \text{ cm (to 1 d.p.)}$$

$$\text{IQR} = 8.5208 \text{ cm}$$

(c) 44.7%

Now try these (p 282)

1. (a) (i) 3 hours

(ii) 11.4 hours

(iii) 7.2 hours

(b) The median is quite low within the IQR, suggesting students generally spent fewer hours studying.

The greatest value is much higher than $Q3$, suggesting a few students (or one student) spent much longer studying than the rest of the class.

Accept other suitable answers.

2. Accept a suitable argument supported by evidence.

Arguments for Swansea:

- most cars sold in one month
- greater median
- median is further along IQR (i.e. there are more higher values)

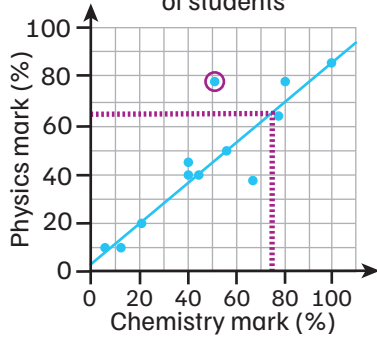
Arguments for Cardiff:

- lower overall range suggests more consistent sales
- greater IQR suggests fewer extreme months
- lowest value is close to $Q1$, suggesting fewer 'bad' months.

(Accept other suitable answers.)

Now try these (p 284)

1. (a) Chemistry and Physics scores of students



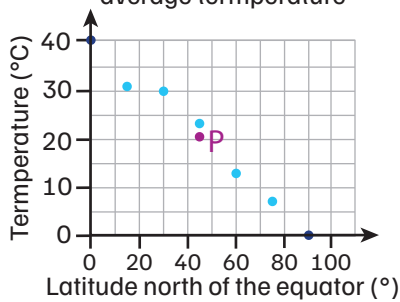
(b) The anomaly is circled in purple.

(c) Weak positive correlation.

(d) Line of best fit is drawn.

Using this line, you can predict that a student with a score of 65% in Physics likely scored 74% in Chemistry.

2. (a) Latitude compared with average temperature



(b) Point *P* has coordinates (45, 20.7) and is labelled above.

(c) There is a strong negative correlation between the latitude north of the equator and temperature.

So, the average temperature decreases moving north from the Equator.

Chapter 4.3 Probability of single events

Now try these (pp 287-288)

1. (a) Likely (d) Even chance
(b) Likely (e) Unlikely
(c) Likely (f) Certain

2. 0.5

3. (a) $P(\text{red}) = \frac{3}{8}$

(b) $P(\text{blue}) = 0.625$

(c) $P(\text{black}) = 0\%$

4. (a) 25

(b) $3x + 2 + x + 1 + 6 = 25$

$$4x + 9 = 25$$

$$4x = 16$$

$$x = 4$$

$$\text{Number of red counters} = 3x + 2$$

$$= 12 + 2$$

$$= 14$$

Now try these (p 290-291)

1.

| Colour | Red | Yellow | Green | Blue |
|--------------------|------|--------|-------|------|
| Relative frequency | 0.35 | 0.25 | 0.15 | 0.25 |

2. (a)

| | | | | |
|--------------------|------|------|------|------|
| Number | 1 | 2 | 3 | 4 |
| Frequency | 8 | 5 | 4 | 3 |
| Relative frequency | 0.40 | 0.25 | 0.20 | 0.15 |

(b)

| Number | 1 | 2 | 3 | 4 |
|--------------------|-----|-----|-----|-----|
| Expected frequency | 780 | 720 | 810 | 690 |

(c) The results from part (a) suggest the spinner is unfair because the relative frequencies are quite different to each other and imply that landing on a 1 is much more likely than landing on a 4

(d) The results from part (b) suggest the spinner is fair because the relative frequencies are more similar to each other.

Chapter 4.4 Probability of multiple events

Now try these (p 293)

1. (a) There are three possible prime numbers

(2, 3, and 5), so the probability is $\frac{3}{6} = \frac{1}{2}$

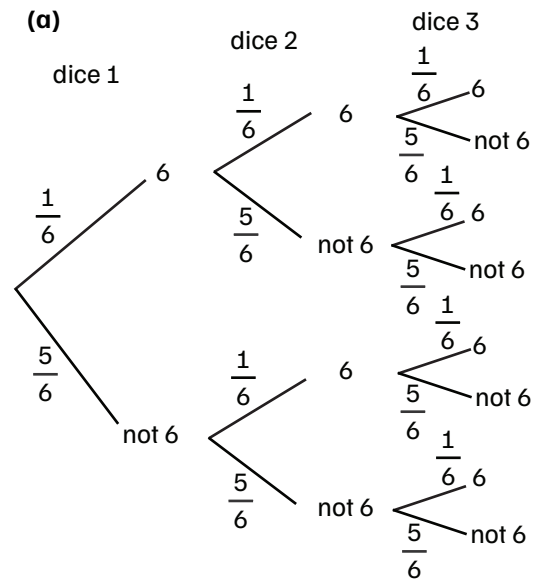
(b) $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

2. (a) Christopher will only order one main meal, meaning he cannot order pasta and pizza at the same time.

(b) $\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$

Now try these (p 296)

1. (a)



- (b) There are four ways that at least two dice can land on a 6:

$$P(1 \text{ and } 2) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216}$$

$$P(2 \text{ and } 3) = \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{5}{216}$$

$$P(1 \text{ and } 3) = \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{5}{216}$$

$$P(1, 2 \text{ and } 3) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$$\text{So, } P(\text{at least two 6s}) = \frac{16}{216} = \frac{2}{27}$$

2. (i) $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

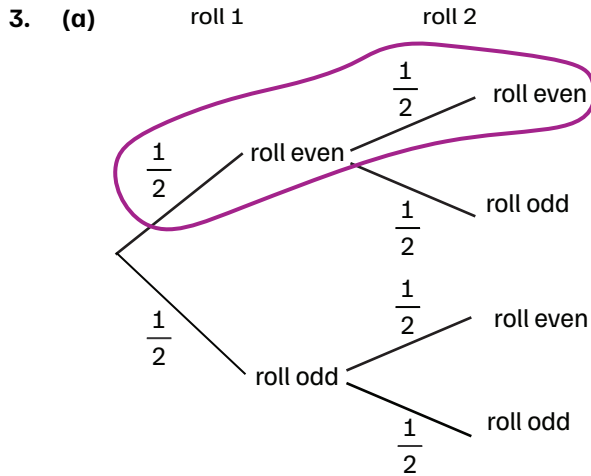
(ii) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{54}$

(iii) $P(RR) = \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$

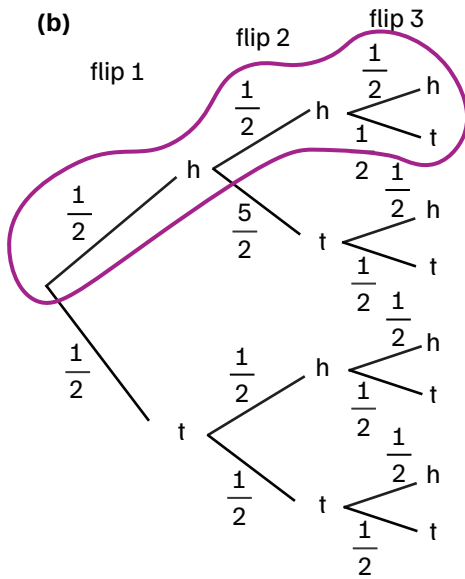
$$P(BB) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(GG) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216}$$

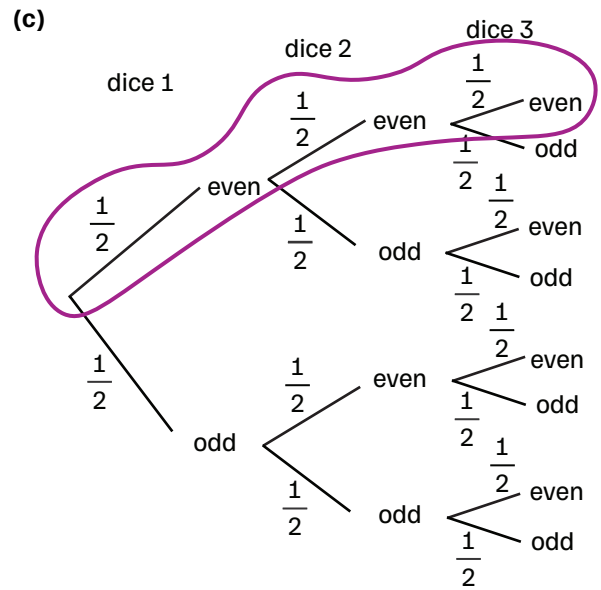
$$\frac{2}{27} + \frac{1}{8} + \frac{5}{216} = \frac{2}{9}$$



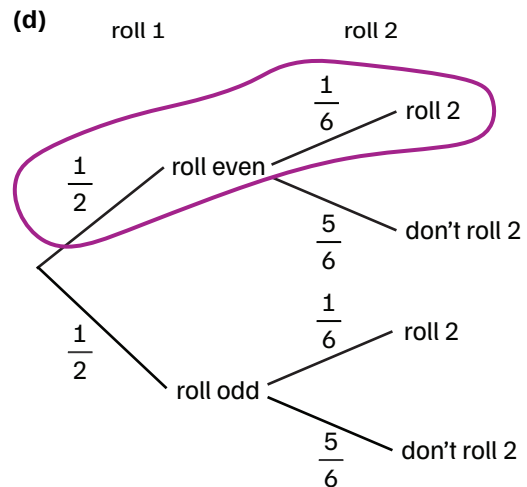
True. The probability of rolling an even number is $\frac{1}{2}$, so the probability of rolling an even number twice is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$



False. The probability of landing on heads three times is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$



False. The probability of rolling an even number is $\frac{1}{2}$, so the probability of rolling an even number three times is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$



False. The probability of rolling an even number is $\frac{1}{2}$ and the probability of rolling a 2 is $\frac{1}{6}$, so the probability of rolling an even number then a 2 is $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Now try these (p 299)

1. (a) 42

(b) 18

(c) 8

(d) $\frac{18}{42} = \frac{3}{7}$

2. (a) 40 students total, so $\frac{14}{40} = \frac{7}{20}$

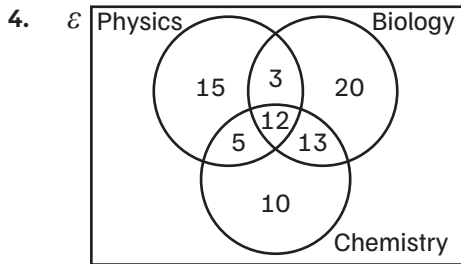
(b) $\frac{15}{40} = \frac{3}{8}$

3. For example:

The diagram is not labelled.

Only 16 students are recorded as having dogs.

There are 31 students on the Venn diagram.



Note that 5 students take Chemistry and Physics only, 3 students take Physics and Biology only (15 overall take Physics and Biology) and 13 students take Chemistry and Biology only (25 overall take Chemistry and Biology).

Now try these (p 301)

1. (a) $P(\text{all blue}) = \frac{12}{20} \times \frac{11}{19} \times \frac{10}{18} = \frac{11}{57}$

$$P(\text{all yellow}) = \frac{8}{20} \times \frac{7}{19} \times \frac{6}{18} = \frac{14}{285}$$

$$P(\text{all same colour}) = \frac{11}{57} + \frac{14}{285} = \frac{23}{95}$$

(b) $P(\text{BY}) = \frac{12}{20} \times \frac{8}{19} \times \frac{7}{18} = \frac{28}{285}$

$$P(\text{YBY}) = \frac{8}{20} \times \frac{12}{19} \times \frac{7}{18} = \frac{28}{285}$$

$$P(\text{YYB}) = \frac{8}{20} \times \frac{7}{19} \times \frac{12}{18} = \frac{28}{285}$$

$$P(\text{one blue chosen}) = 3 \times \frac{28}{285} = \frac{28}{95}$$

(c) $P(\text{one blue chosen}) =$

$$3 \times \left(\frac{12}{20} \times \frac{8}{20} \times \frac{8}{20} \right) = \frac{36}{125}$$

2. (a) $P(\text{all odd}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$

(b) $P(\text{EOO}) = \frac{5}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{5}{36}$

$$P(\text{OEO}) = \frac{5}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{5}{36}$$

$$P(\text{OOE}) = \frac{5}{10} \times \frac{4}{9} \times \frac{5}{8} = \frac{5}{36}$$

$$P(\text{OOO}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$$

$P(\text{no more than one even}) =$

$$\frac{1}{12} + \left(3 \times \frac{5}{36} \right) = \frac{1}{2}$$

3. (a) $P(\text{CCC}) = \frac{13}{20} \times \frac{12}{19} \times \frac{11}{18} = \frac{143}{570}$

(b) $P(\text{VVVVVV}) =$

$$\frac{7}{20} \times \frac{6}{19} \times \frac{5}{18} \times \frac{4}{17} \times \frac{3}{16} \times \frac{2}{15} \times \frac{1}{14} = \frac{1}{77520}$$

Revision round up (p 302–303)

1. (a) Accept any four cards where three of the cards are labelled '8'.

(b) Accept any four cards where none of the cards are labelled with an even number.

2. (a) 60 is a fairly small sample size and none of the values are more than 2 away from 10, so it is possible that these variable frequencies are due to chance rather than bias.

(b) $\frac{12}{60} = 0.2$

(c) $100 \times 0.2 = 20$

3. (a) The students in Shamia's Maths class will probably all be in the same year group.

(b) question 1:

- Add tick box options for neutral / negative opinions.
- Use less subjective language (e.g. 'Do you generally enjoy school lunches?')

question 2:

- Include a time range for answers (e.g. 'How many times a week do you eat school meals?')
- Include a tick box for 0
- Remove overlap of tickboxes (e.g. at the moment, '2' is covered by two tick boxes)

(Accept other suitable answers.)

- (c) If you eat school meals, do you enjoy them?

Always

Usually

About half the time

Not usually

Never

I don't eat school meals

(Accept other suitable answers.)

- (d) Shamia could split the 1200 students in her school into year groups and then use

the formula: $\frac{\text{strata size}}{\text{total population}} \times 50$ to

work out how many people from each year group to include in her sample. If the final sample size is too small or too large, Shamia should then decide which strata should have its number of participants rounded up/down.

4. (a) total = $3 + 12 + 9 + 6 = 30$

median = 15th child = 18 months

$Q1 = 7.5\text{th child} = 14.25$ months

$Q3 = 22.5\text{th child} = 23$ months

(b) $6 \times 0.5 = 3$ children = 10% children

5. Correct method of use of a random number table described, including:

- Assign each student a unique identifier from 01 to 60
- Choose a random number in the table and read in a random direction to generate eight 2-digit numbers
- Ignore 00 and numbers greater than 60
- Use these numbers to select students to include in sample.
- For example, starting with the top left digit and reading right, you could select students 29, 45, 54, 07, 24, 39, 17 and 03

6. Group B, because:

- lower mean (20.4 compared to 21.9 sec)
- lower overall range (6 sec) suggests more consistent sprint times
- lowest sprint time (17 sec)

(Accept other suitable answers.)

7. (a) $P(R-) = \frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$

$$P(-R) = \frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$$

$$P(RR) = \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$$

$$P(\text{at least one red}) = \frac{4}{15} + \frac{4}{15} + \frac{1}{3} = \frac{13}{15}$$

(b) $P(BY) = \frac{3}{10} \times \frac{1}{9} = \frac{1}{30}$

$$P(YB) = \frac{1}{10} \times \frac{3}{9} = \frac{1}{30}$$

$$P(\text{blue and yellow chosen}) = \frac{1}{30} + \frac{1}{30} = \frac{1}{15}$$

(c) $P(RR) = \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$

$$P(BB) = \frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$$

$$P(\text{same colour}) = \frac{1}{3} + \frac{1}{15} = \frac{2}{5}$$

8. Accept a suitable argument for either department, supported by evidence.

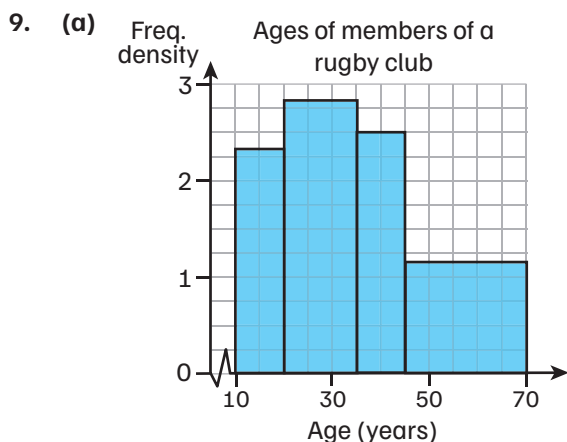
Arguments for department A:

- longer maximum wait time
- greater median
- greater range suggests more inconsistent wait times
- median is closer to Q3 (i.e. there are more higher wait times)

Arguments for department B:

- longer minimum wait time
- wider IQR suggests there is more inconsistency in wait times (ignoring extreme values).

(Accept other suitable answers.)



(b) $345 + 1155 + 1000 + 1725 = 4225$
 $4225 \div 120 = 32.2$ years (to 1 d.p.)

(c) median = 60th person

$$\text{median class} = 20 \leq a < 35$$

$$\text{position of median} = \frac{60 - 23}{42}$$

$$= 88.0\% \text{ along class}$$

$$\text{median} = 20 + (0.8809... \times 15)$$

$$= 33.2 \text{ years (to 1 d.p.)}$$

(d) $20 - 18 = 2$

$$2 \times 2.3 = 4.6 \approx 5 \text{ adults in } 10 \leq a < 20$$

$$5 + 42 + 25 + 30 = 102 \text{ adults in total}$$

$$= 85\% \text{ members}$$

(e) $\frac{42}{120} \times x = 7$

$$x = 7 \div \frac{42}{120} = 20$$

$$10 \leq a < 20: 3.8 \approx 4$$

$$35 \leq a < 45: 4.2 \approx 4$$

$$45 \leq a < 70: 5$$

10. (a) She could check every 20th lightbulb made by the factory.

(b) 4 out of 60 = 6.6...% faulty

$$1200 \times 7 = 8400 \text{ bulbs made per week}$$

$$6.6\% \text{ of } 8400 = 560 \text{ bulbs}$$

(c) No, as 6.6...% < 8%

11. $\frac{6}{n} \times \frac{5}{n-1} = \frac{15}{28}$

$$\frac{30}{n(n-1)} = \frac{15}{28}$$

$$n(n-1) = 56$$

$$n^2 - n = 56$$

$$n = 8$$

12. (a) Accept a suitable argument for either class, supported by evidence.

Arguments for class A:

- minimum score is higher (44% compared to <20%)
- higher Q1 (50% compared to 46%)
- higher Q3 (86% compared to 77%)

Arguments for class B:

- higher median (62% compared to 59%)
- median is about halfway between Q1 and Q3, whereas in class A it is closer to Q1, suggesting a higher proportion of lower marks

(Accept other suitable answers.)

(b) For example:

- the students took the same test
- that marks are evenly distributed within each quartile in class A and are evenly distributed within each class interval in class B. For example, it is possible that the five highest scoring students in class B all scored 80% or all scored 100%.

(Accept other suitable answers.)

(c) Cumulative frequency at 70% is 12.5

$12.5 \div 20 = 62.5\%$ students scored below 70%, so 37.5% scored higher.

However, 37.5% of 20 students isn't possible, so round down.

35% students scored higher than 70%.

$$Q1 = 3 + (0.628... \times 9)$$

$$= 8.7 \text{ months (to 1 d.p.)}$$

$$Q3 = 337.5\text{th sheep}$$

$$Q3 \text{ class} = 24 \leq a < 48$$

$$\text{position of } Q3 = \frac{337.5 - 294}{156}$$

$$= 27.8... \% \text{ along class}$$

$$Q3 = 24 + (0.278... \times 24)$$

$$= 30.7 \text{ months (to 1 d.p.)}$$

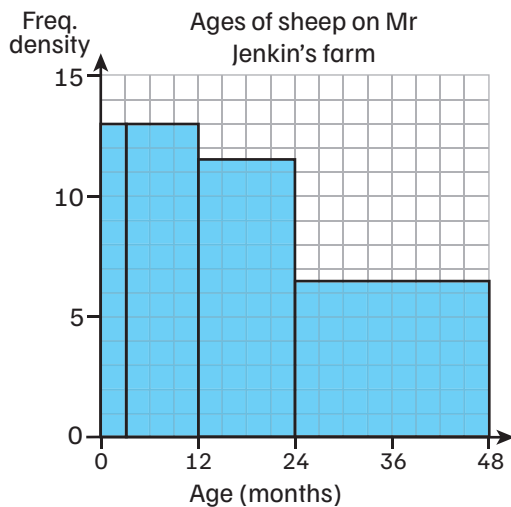
So, IQR = 22 months

(c) Number of sheep in $3 \leq a < 12$ class that are over 9 months old = $13 \times 3 = 39$

Total number of sheep over 9 months old = $39 + 138 + 156 = 333$ sheep

= 74% sheep

13. (a)



(b) total = 450 sheep

median = 225th sheep

median class = $12 \leq a < 24$

$$\text{position of median} = \frac{60 - 23}{42}$$

$$= 50\% \text{ along class}$$

$$\text{median} = 12 + (0.5... \times 12)$$

$$= 18 \text{ months}$$

Q1 = 112.5th sheep

Q1 class = $3 \leq a < 12$

$$\text{position of } Q1 = \frac{112.5 - 39}{117}$$

$$= 62.8... \% \text{ along class}$$

Revision practice questions

Unit 1

Calculator (pp 304–307)

1. (a) 1.04×10^4
(b) 5×10^{11}
(c) 5×10^2
2. (Electricity cost is) 660×0.29 or 660×29
= (£) 191.4(0) or 19 140 (p)
(Standing charge + electricity is)
(£) 236.4(0) or 23 640 (p)
(Total bill including VAT is)
 $1.05 \times 236.4(0)$ or $1.05 \times 23\ 640$
= (£) 248.22 or 24 822 (p)
3. 28.8 minutes
4. (a) $45 \times \text{£}11.44 = \text{£}514.80$ a week
(b) $45 \times \text{£}12.21 = \text{£}549.45$ a week
(c) % increase = $\frac{\text{increase}}{\text{original}} \times 100$
 $= \frac{(549.45 - 514.80)}{514.80} \times 100$
 $= 6.7\%$ to 1 d.p.
5. (a) $\frac{1}{8}$
(b) $4a^{10}b^{-7}$
6. small = 2.2105...p per ml
medium = 1.16p per ml
large = 1.24...p per ml
So, the medium bottle is the best buy.
7. (a) loan A:
 $r = 7.5\% \div 12 = 0.625\% = 0.00625$
 $n = 12 \times 3 = 36$
 $M = \frac{0.00625 \times \text{£}15\ 000}{1 - (1 + 0.00625)^{-36}}$
 $M = \text{£}466.59$ (2 d.p.)
Total amount paid = $\text{£}466.59 \times 36$
 $= \text{£}16\ 797.24$
loan B:
 $r = 9\% \div 12 = 0.75\% = 0.0075$
 $n = 12 \times 2 = 24$
 $M = \frac{0.0075 \times \text{£}15\ 000}{1 - (1 + 0.0075)^{-24}}$
 $M = \text{£}685.27$ (2 d.p.)
Total amount paid = $\text{£}685.27 \times 24$
 $= \text{£}16\ 446.48$
So, loan B results in the least amount paid back
- (b) loan A
8. (a) $i = 5.6\% \div 100 = 0.056, n = 4$
 $\text{AER} = \left(1 + \frac{0.056}{4}\right)^4 - 1$
 $\text{AER} = 5.72\%$
(b) Banks and societies use AER to compare the amount of interest that would be generated from different savings accounts over one year, accounting for things like different compounding rates.
9. $x = -1$
10. (a) 17.8 (to 3 s.f.)
(b) $-\frac{5}{3} = -1.666\dots = -1.7$ (to 1d.p.)
11. (a) $\text{£}5600$
(b) $\text{£}13\ 100 - \text{£}5600 = \text{£}7500$ paid at basic rate
 $\text{£}7500 \div 0.2 = \text{£}37\ 500$ taxable income within basic band
 $\text{£}50\ 000 - \text{£}37\ 500 = \text{£}12\ 500$
So, personal allowance = $\text{£}12\ 500$
(c) $\text{£}3750 \times 12 = \text{£}45\ 000$ per year

$£45\,000 - £12\,500 = £32\,500$ taxable
 income at basic rate
 $£32\,500 \times 0.2 = £6\,500$ income tax paid
 $£13\,100 - £6\,500 = £6\,600$ more paid

12. 6 children

20 teenagers

14 young adults

13. (a) price during winter = $£5 \times 1.45 = £7.25$

$$\begin{aligned} \text{price during sale} &= £7.25 \times \frac{2}{3} \\ &= £4.83 \text{ (to 2 d.p.)} \end{aligned}$$

So, Eilir loses 17p for each pair.

(b) $£5 = £7.25 \times x$

$$x = 0.6896\dots = 68.96\dots\%$$

So, smallest discount = $100\% - 68.96\dots\%$
 $= 31\%$ (nearest %)

14. (a) 121°F

(b) 63°C

(c) No, because the graph does not pass through the origin.

15. Wales = 6

Rest of GB = 18

Rest of world = 8

16. Let l = length of bottom cup and x = distance between tops of two stacked cups.

$$l + x = 19$$

$$l + 4x = 34$$

$$3x = 15, \text{ so } x = 5$$

$$l + 5 = 19 \text{ so } l = 14 \text{ cm}$$

17. (a) Accept any suitable answer. For example:

- random number generator
- random number table
- lottery system (e.g. names in a hat)

(b) 11 Year 10 students

9 Year 11 students

(c) The headmaster should obtain a list of all Year 10 and 11 students and select every

6th student to be in their sample.

(d) Accept any suitable answer supported by explanation. For example:

- random sampling: simple to carry out and everyone has an equal chance of being selected
- systematic sampling: simple and fast to carry out
- stratified sampling: most representative as it takes into account the differing number of Year 10 and Year 11 students

18. Rhodri = 8 years old

Ally = 16 years old

Megan = 48 years old

19. (a) upper bound of circumference = 69.5 cm

$$\text{circumference} = 2\pi r$$

$$\text{upper bound of radius} = 69.5 \div 2\pi$$

$$= 11.06 \text{ cm (to 2 d.p.)}$$

$$(b) V = \frac{4}{3}\pi r^3 = 5668.96 \text{ cm}^3 \text{ (to 2 d.p.)}$$

20. (a) top = $\pi r^2 = 4\pi \text{ m}^2$

$$\text{side} = 2\pi r h = 16\pi \text{ m}^2$$

$$\text{total SA} = 20\pi \text{ m}^2$$

$$(b) V = \pi r^2 h = 16\pi \text{ m}^3$$

21. (a) $£5500 \times 1.002^{36} = £5910.18$ (to 2 d.p.)

$$\text{total interest paid} = £410.18 \text{ (to 2 d.p.)}$$

(b) $£5500 \times 1.002^{24} = £5770.16$ (to 2 d.p.)

$$\text{Aled would save } £140.02$$

22. £562.50

23. (a) 100 000 000 mm²

$$(b) 7\,500\,000\,000 \text{ m}^3$$

$$(c) 0.00000000012 \text{ km}^2$$

24. (a) 5.25 days

(b) 21 workers

(c) Assuming that every worker works at the same rate

Unit 2

Non-calculator (pp 307–310)

- (a) True
(b) True
(c) True
(d) False. Dividing by 10 and then by 2 is the same as dividing by 20
(e) True
- (a) exterior angle = $\frac{360}{5} = 72^\circ$
(b) $(n - 2) \times 180 = 900^\circ$
 $n = 7$ sides
- (a) t
(b) 2
(c) $\frac{3x}{y}$
(d) $100a^4b^4$
- (a) $4100 \times 0.08 (= 328 \text{ and}) 41 \times \frac{1}{8} = 328$
(b) $8^{\frac{2}{3}} (= 4 \text{ and}) \frac{1}{0.25} = 4$
- (a) $-12n + 1107$
(b) $-12n + 1107 < 50$
 $n > 88.083\dots$
so, the 89th term (39) is the first term that is less than 50
(c) $-3n^2 + 64$
(d) $-3n^2 + 64 < -25$
 $n > 5.446\dots$
so, the 6th term (-44) is the first term that is less than -25
- (a) $3(x + 5)(x - 3)$
(b) $\frac{2x^2 - 7x - 15}{4x^2 - 9} = \frac{(2x + 3)(x - 5)}{(2x + 3)(2x - 3)}$
 $= \frac{x - 5}{(2x - 3)}$
- (a) $3 \leq x < 7$
(b) 1, 2 and 3
- (a) $x = 10^\circ$
(b) $8x = 80^\circ$
- A: true
B: true
C: false
D: true
E: true
F: false
- (a) $5(2x - y) = 30 \text{ cm}$
 $3(x + y) = 18 \text{ cm}$
 $x = 4 \text{ cm}, y = 2 \text{ cm}$
(b) $2x - y = 6 \text{ cm}$
(c) $x + y = 6 \text{ cm}$
- $\angle ABC = 70^\circ$ (alternate segment theorem)
 $\angle BAC = 180^\circ - 65^\circ - 70^\circ = 45^\circ$ (angles in a triangle add to 180°)
 $\angle BAQ = 180^\circ - 70^\circ - 45^\circ = 65^\circ$ (angles on a straight line add to 180°) or (alternate segment theorem)
- (a) 3 100 000 people
(b) 20 780 km^2
(c) 1700 miles
(d) $3\,000\,000 \div 20\,000 = 150 \text{ people/km}^2$
- (i) 1
(ii) -2
(iii) 6
- (a) $x = 9$
(b) $x = \frac{1}{2}$
(c) $x = \frac{1}{2}$
- $g^3h^3 = 27g^3 + 7$ or $g^3h^3 - 27g^3 = 7$
 $g^3(h^3 - 27) = 7$
 $g^3 = \frac{7}{h^3 - 27}$
 $g = \sqrt[3]{\frac{7}{h^3 - 27}}$
- (a) $m = 2$
(b) $y - 2x + 8 = 0$ and $2y = 4x - 12$

17. (a) 21:30 Sydney time

(b) 21:35 Sydney time

18. $\sqrt{11}$

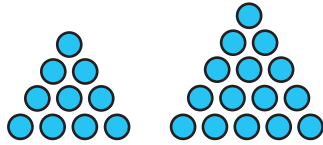
19. $x = 3$, perimeters = 27 cm

20. (a) $x = 360^\circ - 90^\circ - 90^\circ - 40^\circ$
 $= 140^\circ$ (360° in a quadrilateral,
the tangent at any point is always
perpendicular to the radius at that
point.)

$y = 140^\circ \div 2 = 70^\circ$ (angle subtended at
circumference is half angle subtended at
origin by same arc.)

(b) $\angle ABC = 180^\circ - \angle CDA$ (opposite angles in
a cyclic quadrilateral add to 180°)
 $\angle DAB = 180^\circ - \angle BCD$ (opposite angles in
a cyclic quadrilateral add to 180°)
so, $\angle DAB = \angle ABC$
so, $\angle CDA + \angle DAB = 180^\circ$, meaning they
are co-interior angles and lines DC and
AB are parallel.
So, the shape has one pair of parallel
sides and is therefore a trapezium.

21. (a)



(b) 21

(c) $n^2 + 4n - 3$

22. $27\,002 \approx 27\,000$, $8.05 \approx 8$

$$\sqrt[3]{\frac{27\,000}{8}} = \frac{30}{2} = 7.5$$

23. (a) $\frac{9}{200}$ or equivalent

(b) $\frac{9}{200} \times 5000$
 $= 225$

24. $(x + 2)(2x + 1)$
 $x = -2$ or $x = -\frac{1}{2}$

25. $\frac{181}{333}$

26.



regular ticket = £30

early bird ticket = £20

27. (a) graph 1: $y = (ax + c)(bx + d)$

graph 2: $y = ax^2 + b$

graph 3: $y = x^3 + b$

graph 4: $y = \frac{a}{x}$

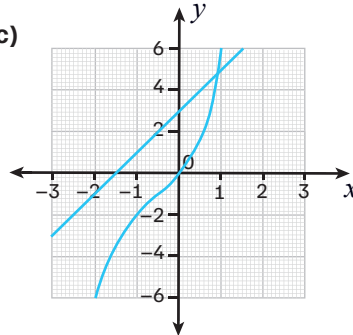
(b) graph 1: $(-\frac{c}{a}, 0), (-\frac{d}{b}, 0), (0, cd)$

graph 2: $(0, b)$

graph 3: $(0, b), (\sqrt[3]{-b}, 0)$

graph 4: no points of intersection

(c)



$(0.9, 4.7)$

28. (a) $P(\text{RBY}) = \frac{13}{20} \times \frac{5}{19} \times \frac{2}{18} = \frac{13}{684}$

There are six different ways you could
choose three different-coloured counters
(RBY, RYB, BRY, BYR, YBR, YRB), so:

$$P(\text{3 diff colours}) = \frac{13}{684} \times 6 = \frac{13}{114}$$

(b) The only scenario where two counters
aren't the same colour is where all three
counters are different colours.

$$\text{So, } P(\text{at least 2 the same}) = 1 - \frac{13}{114}$$

$$= \frac{101}{114}$$

$$(c) P(RRR) = \frac{13}{20} \times \frac{12}{19} \times \frac{11}{18} = \frac{143}{570}$$

$$P(BBB) = \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{1}{114}$$

(cannot have 3 yellow counters)

$$P(3 \text{ the same}) = \frac{143}{570} + \frac{1}{114} = \frac{74}{285}$$

$$(d) P(\text{not getting a yellow}) = \frac{18}{20}$$

$$P(\text{not yellow 9 times}) = \left(\frac{18}{20}\right)^9 = 0.39 \text{ (to 2 d.p.)}$$

$$29. \frac{2a+3}{3} = \frac{4a+1}{5}$$

$a = 6$, so Isabel is 6 and Thomas is 12

Unit 3

Calculator (pp 310–313)

$$1. \text{ Area of square} = \sqrt{6} \times \sqrt{6} = 6 \text{ cm}^2$$

$$\begin{aligned} \text{Area of semicircles} &= 2 \times \left(\frac{1}{2} \times \pi \times r^2\right) \\ &= \frac{3}{2} \pi \text{ cm}^2 \end{aligned}$$

$$\text{Area of shaded region} = \left(6 - \frac{3}{2} \pi\right) \text{ cm}^2$$

$$2. (a) \tan 37^\circ \times 27 = 20.35 \text{ m (to 2 d.p.)}$$

$$(b) \tan^{-1}\left(\frac{52}{27}\right) = 62.6^\circ \text{ (to 1 d.p.)}$$

$$3. (a) a^2 - 4ab + 4b^2$$

$$(b) 2a^3 + 7a^2 - 5a - 4$$

$$4. (a) \text{ area} = (2x+1)(x+3)$$

$$= 2x^2 + 7x + 3$$

$$(b) 2x^2 + 7x + 3 = 42$$

$$2x^2 + 7x - 39 = 0$$

$$x = 3$$

$$\text{perimeter} = 2(6+1) + 2(6) = 26 \text{ cm}$$

$$5. a = 3, b = -1$$

$$6. (a) \text{ when } x = 5, x^3 - 35x + 10 = -40$$

$$\text{when } x = 6, x^3 - 35x + 10 = 16$$

0 lies between -40 and 16 , so the solution lies between 5 and 6

(b) using iterative formula $x = \sqrt[3]{35x-10}$
and starting value $x = 5.5$:

$$x = 5.7676\dots = 5.77 \text{ (to 2 d.p.)}$$

$$7. (a) AC = x + 3 \text{ cm}$$

$$\frac{1}{2}(x+3)(x+4) = 21$$

$$x = 3 \text{ cm}$$

$$(b) BC^2 = (x+3)^2 + (x+4)^2$$

$$BC^2 = 6^2 + 7^2$$

$$BC^2 = 85$$

$$BC = \sqrt{85} = 9.22 \text{ (to 3 s.f.)}$$

8. (a) $\angle ACB = \angle ECD$ (opposite angles are equal)

$\angle BAC = \angle CDE$ (alternate angles are equal)

$\angle ABC = \angle CED$ (alternate angles are equal)

$$(b) 10 \div 5 = 2$$

$$(c) 24 \div 2 = 12 \text{ cm}$$

$$(d) 4.5 \times 2 = 9 \text{ cm}$$

9. A: false. $y = \cos x$ does not pass through the point $(0, 0)$.

B: true

C: false. $y = \sin x$ does not pass through the point $(90, 0)$.

D: true

E: true

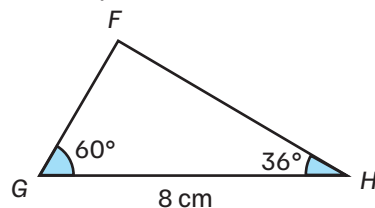
10. $PQ = PR$ (given)

$PX = PY$ (midpoints of PR and PQ)

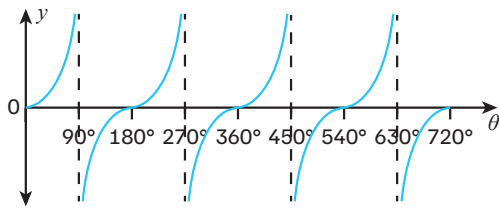
$\angle RPY = \angle XPQ$ (shared angle)

So, triangles PRY and PQX are congruent under the SAS condition.

11. (check exact measurements using a ruler and protractor)



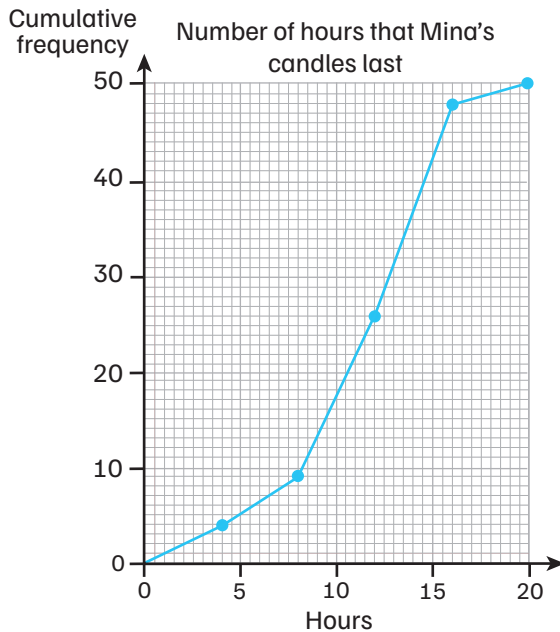
12. (a)



(b) $153^\circ, 333^\circ, 513^\circ, 693^\circ$ to the nearest degree.

13. 5.2%

14. (a)



(b) 11.75 hours

(c) four out of five = 40 out of 50 candles

As shown in the graph, the 10th candle lasted about 8.25 hours, so the 40 candles that lasted the longest lasted for at least 8.25 hours.

So, Mina is wrong, as four out of five of her candles last longer than 8.25 hours, not 10

15. (a) $51\,839 - 40\,981 = 10\,858$ miles

(b) $10\,858 \div 100 = 109$ charges

(c) $109 \times \text{£}3 = \text{£}327$

16. (a) scale factor = $8 \div 12 = \frac{2}{3}$

$$9 \times \frac{2}{3} = 6 \text{ cm}$$

(b) linear scale factor = $\frac{2}{3}$

$$\text{area scale factor} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\text{area of } P = 12\sqrt{3} \div \frac{4}{9} = 27\sqrt{3} \text{ cm}^2$$

17. $\angle ABC = \angle DEC$ (corresponding angles are equal)

$\angle BAC = \angle EDC$ (corresponding angles are equal)

$\angle ACB = \angle DCE$ (shared angle)

So, triangles ABC and CDE are similar.

18. (a) $12^2 = 8^2 + 17^2 - 2(8)(17)(\cos A)$

$$A = 39.7911\dots^\circ$$

$$\frac{12}{\sin 39.7911\dots} = \frac{8}{\sin B} = \frac{17}{\sin C}$$

$$B = 25.2558\dots^\circ$$

$$C = 65.0469\dots^\circ$$

However, the angle where the two shortest sides meet needs to be obtuse for the interior angles to add to 180° .

This is angle C .

Using sine graph:

$$C = 65.0469\dots^\circ \text{ or } C = 180^\circ - 65.0469\dots^\circ = 114.9531\dots^\circ$$

So, the angles are $39.8^\circ, 25.3^\circ$ and 115.0° (all to 1 d.p.)

$$\begin{aligned} \text{(b) area} &= \frac{1}{2} \times 12 \times 8 \times \sin 114.9531\dots^\circ \\ &= 43.52 \text{ cm}^2 \text{ (to 2 d.p.)} \end{aligned}$$

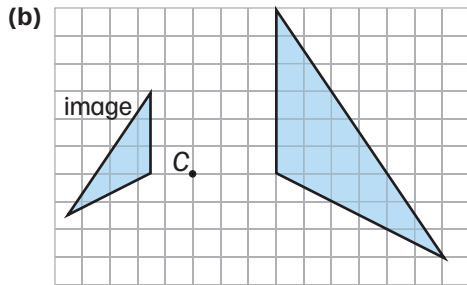
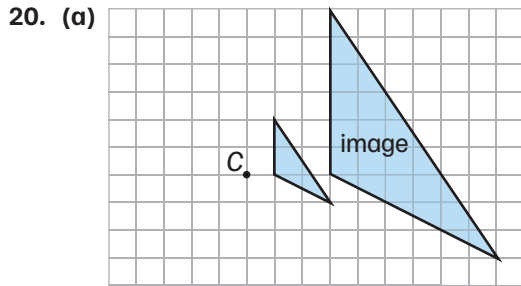
19. (a) volume = $(3 \times 3 \times 3) + (2 \times 3 \times 2)$
= 39 units cubed

$$\begin{aligned} \text{(b) SA of whole cuboid} &= 2(3 \times 3) + 2(3 \times 5) \\ &\quad + 2(3 \times 5) \\ &= 78 \text{ units squared} \end{aligned}$$

$$\begin{aligned} \text{SA of missing portions} &= 2(2 \times 1) \\ &= 4 \text{ units squared} \end{aligned}$$

$$\begin{aligned} \text{SA of whole shape} &= 78 - 4 \\ &= 74 \text{ units squared} \end{aligned}$$

(c) SA of whole cuboid = $2(3 \times 3) + 2(3 \times 5) + 2(3 \times 5)$
 $= 78$ units squared
 SA of missing portions = $2(2 \times 2)$
 $= 8$ units squared
 SA of whole shape = $78 - 8$
 $= 70$ units squared



21. $l = \frac{15}{360} \times 2 \times \pi \times 0.9 = 0.24$ cm (to 2 d.p.)

22. $V(\text{large cone}) = \frac{1}{3} \pi r^2 h$
 $= 252\pi$ cm³

linear scale factor = $9 \div 21 = \frac{3}{7}$

r of smaller cone = $6 \times \frac{3}{7} = \frac{18}{7}$ cm

$V(\text{small cone}) = \frac{1}{3} \pi r^2 h$
 $= \frac{972}{49} \pi$ cm³

$V(\text{frustum}) = 252\pi - \frac{972}{49} \pi$
 $= \frac{11376}{49} \pi$ cm³

23. Finding the appropriate angle 108°
 (Distance between Manchester and Lisbon =)

$$\sqrt{605^2 + 1440^2 - 2 \times 605 \times 1440 \times \cos 108}$$

$= 1725.7(045\dots)$ (km)

$12 \times (605 + 1440 - 1725.7)$ or equivalent
 An answer between 3831 and 3832 (litres) inclusive

24. (a) $250 < x \leq 350$

(b) Midpoints 100, 200, 300, 400

$$100 \times 23 + 200 \times 84 + 300 \times 116 + 400 \times 28$$

$$= 2300 + 16800 + 34800 + 11200$$

$$= 65100$$

$$\div 251$$

259.3(6..) miles or 259.4 or 259 miles or equivalent

25. (a) the bike was speeding up

(b) the bike was stationary

(c) using tangent that passes through points (6, 6) and (15, 9.5), gradient = 0.388...

However, acceleration is given in km/h² or m/s², so the x -axis values need to be converted to hours or the y -axis values need to be converted to m/s.

$$\text{So, acceleration} = \frac{9.5 - 6}{\frac{15}{3600} - \frac{6}{3600}}$$

$$= 1400 \text{ km/h}^2$$

or acceleration =

$$\left(9.5 \times \frac{1000}{3600} \right) - \left(6 \times \frac{1000}{3600} \right)$$

$$\frac{\quad}{15 - 6}$$

$$= 0.11 \text{ m/s}^2 \text{ (to 2 d.p.)}$$

(d) Using trapezium rule with 6 strips:

$$\text{Area} = \frac{1}{2} (5 + 7.5) \left(\frac{10}{3600} \right)$$

$$+ \frac{1}{2} (7.5 + 14) \left(\frac{10}{3600} \right)$$

$$+ \frac{1}{2} (14 + 5) \left(\frac{10}{3600} \right)$$

$$+ \frac{1}{2} (5 + 0) \left(\frac{10}{3600} \right)$$

$$+ \frac{1}{2} (0 + 0) \left(\frac{10}{3600} \right)$$

$$+ \frac{1}{2} (0 + 5) \left(\frac{10}{3600} \right)$$

$$= 0.0875 \text{ km} = 87.5 \text{ m travelled}$$

26. area = $\left(\frac{72}{360} \times \pi \times 3^2 \right) - \left(\frac{1}{2} \times 3^2 \times \sin 72^\circ \right)$
 $= 1.38$ cm² (to 2 d.p.)

27. (a) $EG = 10$ cm

$$GD = \sqrt{9^2 + 12^2} = 15 \text{ cm}$$

$$ED = \sqrt{10^2 + 15^2} = 5\sqrt{13} = 18.03 \text{ cm (to 2 d.p.)}$$

(b) $EH = \sqrt{10^2 + 9^2} = \sqrt{181}$ cm

$$(\sqrt{181})^2 = (5\sqrt{13})^2 + 12^2 - 2(5\sqrt{13})(12)(\cos \angle EDH)$$

$$\angle EDH = 48.2685\dots^\circ$$

$$\text{area} = \frac{1}{2} \times 5\sqrt{13} \times 12 \times \sin 48.2685\dots^\circ$$

$$= 80.72 \text{ cm}^2 \text{ (to 2 d.p.)}$$

(c) $\frac{\sqrt{181}}{\sin 48.2685\dots} = \frac{12}{\sin \angle DEH}$

$$\angle DEH = 41.73^\circ \text{ (to 2 d.p.)}$$

28. (a) $p = \frac{0.2}{q^2}$ or $p = \frac{1}{5q^2}$

(b) $2\sqrt{5}$

29. (a) Rotate 90° clockwise using a centre of rotation of $(3, 1)$. Then, translate 4 units right and 3 units up.

(Accept other suitable answers.)

(b) Rotate 90° clockwise using a centre of rotation of $(6.5, 0.5)$.

(c) Enlarge by a scale factor of 3 using a centre of enlargement of $(12, 7)$. Then, reflect in the line $y = 5$

(Accept other suitable answers.)

(d) Enlarge by a scale factor of -3 using a centre of enlargement of $(12, 6)$.